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On the numerical solution of constrained eigenvalue problems in structural engineering

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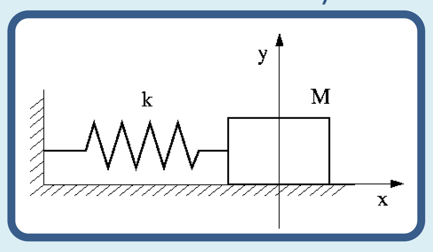
The modal analysis

The modal analysis is the study of the dynamic properties of structures under vibrational excitation.

The free vibration equilibrium equations are

$$K\ddot{U} + MU = 0 \qquad \text{(FVE)}$$

$U \in \mathbb{R}^n$ is the Finite Element (FE) displacement vector, $K, M \in \mathbb{R}^{n\times n}$ are the stiffness and mass matrix of the FE assemblage, n is the total number of dofs of the system.



The solution of (FVE) can be assumed to be of the form

$$U(t) = u\,\sin(\omega(t - t_o))$$

$u \in \mathbb{R}^n$, t is the time variable, t_0 a time constant, ω is a constant that represents the frequency of vibration (rad/sec) of the vector u.

Substituting $U(t)$ in (FVE), we obtain the generalized eigenproblem (GEP)

$$Ku = \omega^2 Mu$$

The solution of (GEP) allows to find the various periods at which a structure naturally resonates.

The multi-point constraints

Fixed (single-point) constraints

$$u_i = 0, i \in I_F$$

where u_i is the displacement of a single dof.

Master-Slave constraints or tying relations: there exists a subset $I_S \in \{1, \dots, n\}$ such that

$$u_s = \sum_{m\in I_{M_S}} c_{sm} u_m, \quad s \in I_S,\ m \in I_{M_S} \subset \{1, \dots, n\} \setminus I_S$$

u_s is the slave dof, u_m are the master dofs.

These constraints are crucial in modeling the contact interaction between masonry and reinforcement.

The constrained eigenvalue problem

$$Ku = \omega^2 Mu \ s.t.\ Cu = 0$$

$K, M \in \mathbb{R}^{n\times n}$ are sparse and symmetric positive semidefinite, $C \in \mathbb{R}^{m\times n}$ is full row rank and K is positive definite on the $null(C)$.

Let the columns of C be ordered so that $C = [C_{I_F} C_{I_S} C_{I_M} C_{I_U}]$, then the matrix

$$Z = \begin{pmatrix} -[C_{I_M} 0] \\ Id_{n-m} \end{pmatrix} \in \mathbb{R}^{n\times(n-m)}$$

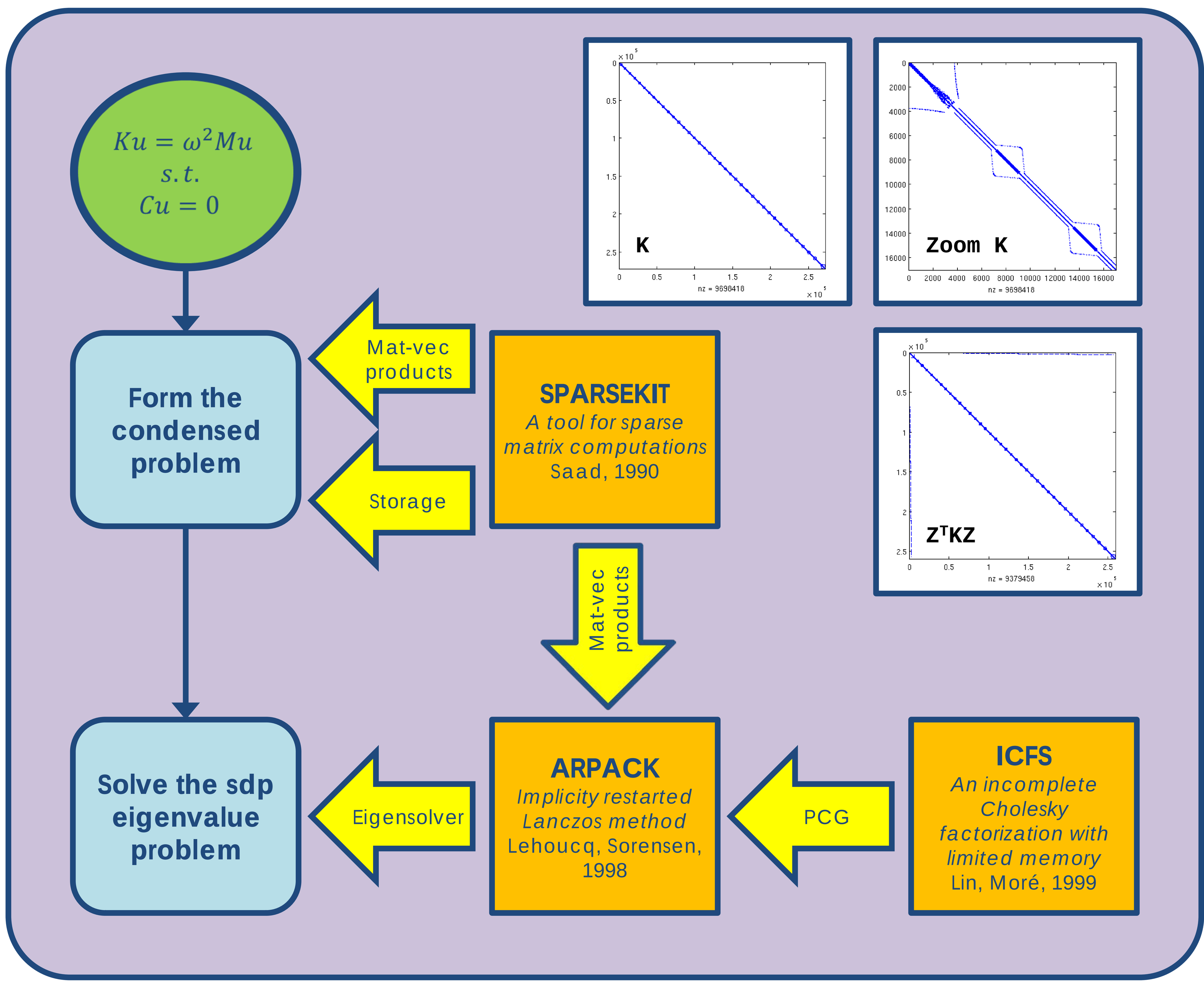
forms a basis for the $null(C)$, where I_F, I_S, I_M, I_U denote the sets of fixed, slave, master and free dofs.

The condensed eigenvalue problem

Find the $s \ll n$ smallest eigenpairs of

$$Z^T K Z\, u = \omega^2 Z^T M Z$$

If K, M are sparse $\Rightarrow$ the projected matrices $Z^T K Z$ and $Z^T M Z$ are sparse ($m \ll n$).



Case study: the "Voltone", Piazza della Repubblica, Livorno, Italy

The "Voltone" is a large underground vaulted masonry structure (thick 0,43 meters) located beneath Piazza della Repubblica in Livorno, Italy; it extends for a total length of 220 meters and is constituted by two longitudinal walls (thick 2,32 meters) with variable height, on which the vault is placed. The aim of the research is the application of the NOSA-ITACA code for assessing its static safety and seismic vulnerability; in particular, the modal analysis is used for a preliminary study on the dynamic response of the structure, as recommended by the Italian Technical Regulations.

The structure's geometry was obtained by a 3D digital survey. The finite element mesh is constituted by 43084 thick shell elements and by 45379 nodes. The modal frequencies calculated by NOSA-ITACA mainly involve the vaulted structure, which is much more slender than the lateral walls.

The activity is carried-out within the project "Tools for the modelling and assessment of the structural behaviour of ancient construction: the NOSA-ITACA code", funded by Region of Tuscany, 2011-2013.

www.nosaitaca.it