Analisi numerica di problemi di ingegneria strutturale

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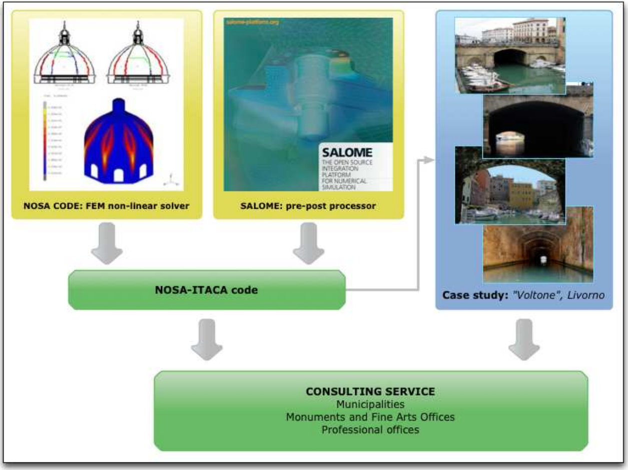
The NOSA-ITACA project (2011-2013, Cultural Heritage)

- Partners: ISTI -CNR Research Area of Pisa
  Department of Constructions and Restoration - UNIFI
- Application: to support the activities of safeguarding and strengthening historic masonry constructions.
- Numerical analysis: methods for large constrained eigenvalue problems.
- Funded by PAR FAS Regione Toscana

Tools for the modelling and assessment of the structural behaviour of ancient masonry constructions: the NOSA-ITACA code



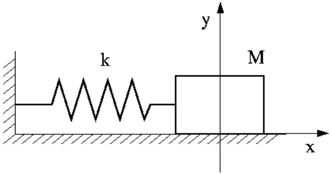
The modal analysis

- The modal analysis is the study of the dynamic properties of structures under vibrational excitation.

Free vibration equilibrium equations with dumping neglected

$$M\ddot{U} + KU = 0$$

- $U \in \mathbb{R}^n$ is the FE displacement vector (time dependent).
- $K, M \in \mathbb{R}^{n \times n}$ are the stiffness and mass matrix of the FE assemblage.
- $n =$ total number of degrees of freedom (DOF) of the system.



- The solution of the equilibrium equation can be postulated to be of the form
$$U = u \sin(\omega(t - t_0)),$$
  - $u \in \mathbb{R}^n$, t is the time variable, t_0 a time constant,
  - ω a constant that represents the frequency of vibration (rad/sec) of the vector u.

Find the $s \ll n$ smallest eigenpairs of the generalized eigenproblem (GEP)
$$K\,u = \omega^2\,M\,u$$

- The solution of the (GEP) allows to find the various periods at which a structure naturally resonates.
- The number of frequencies of interest is mainly related to the geometry of the structure and its mass distribution.

Constraints: $C\,u = 0$, $C \in \mathbb{R}^{m \times n}$, $m \ll n$

Fixed (single-point) constraints

$$u_i = 0, i \in I_F$$

where u_i is the displacement of a single DOF (e.g. imposition Dirichlet boundary conditions).

Master-Slave (multi-points) constraints or tying relations

There exists a subset $I_S \subset \{1, \ldots, n\}$ such that

$$u_s = \sum_{m \in I_{M_s}} c_{sm} u_m, \quad s \in I_S, m \in I_{M_s} \subset \{1, \ldots, n\} \setminus I_S$$

u_s is the slave DOF whereas u_m are the master DOFs.

- These constraints are crucial, e.g., in modeling the contact interaction between masonry and reinforcement.

The constrained eigenvalue problem

$$K\,u = \lambda\, M\,u \quad \text{subject to } C\,u = 0$$

$K, M \in \mathbb{R}^{n\times n}$ pos. semidef. sparse and symmetric, $C \in \mathbb{R}^{m\times n}$ full row rank and K is pos. def. on the $Null(C)$.

Unconstrained reformulations

- Condensed problem: let $Z \in \mathbb{R}^{n\times(n-m)}$ be an orthonormal matrix whose columns span $null(C)$,

  $$(Z^T KZ)\,v = \lambda\,(Z^T MZ)\,v, \quad v = Z^T u \in \mathbb{R}^{(n-m)}$$

  [Hitziger,Mackens,Voss 1995]

- Augmented problem:

  $$\begin{bmatrix} K & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix} = \lambda \begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}, y \in \mathbb{R}^m.$$

  [Baker, Lehoucq 2007]

Forming a basis for *null*(C)

Lemma

Let $C \in \mathbb{R}^{m\times n}$ a full row rank matrix and let $C = GH$ a rank-retaining factorization, i.e. let $G \in \mathbb{R}^{m\times m}, H \in \mathbb{R}^{m\times n}$ with $rank(G) = rank(H) = m$.

Moreover, let H be partitioned as

$$H = [H_m \; H_{n-m}]$$

$$H_m \in \mathbb{R}^{m\times m} \text{ non singular and } H_{n-m} \in \mathbb{R}^{m\times(n-m)}.$$

Then, the columns of the matrix $Z \in \mathbb{R}^{n\times(n-m)}$ given by

$$Z = \begin{bmatrix} -H_m^{-1} H_{n-m} \\ I_{n-m} \end{bmatrix}$$

form a basis for *null*(C).

Results for Master-Slave constraints

Let the columns of C be ordered so that $C = [C_{I_F}\, C_{I_S}\, C_{I_M}\, C_{I_U}]$, then

$$C = GH, \text{ with } G = I_m, H = C$$

is a rank-retaining factorization and

$$H_m = [C_{I_F} \;\; C_{I_S}] \text{ and } H_{n-m} = [C_{I_M} \;\; 0].$$

Then

$$Z = \begin{bmatrix} -[C_{I_M} \;\; 0] \\ I_{n-m} \end{bmatrix}$$

- If K, M are sparse $\Rightarrow$ the projected matrices $Z^T K Z, Z^T M Z$ are sparse $(m \ll n)$.

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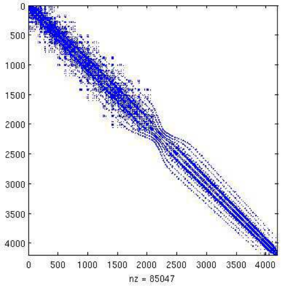
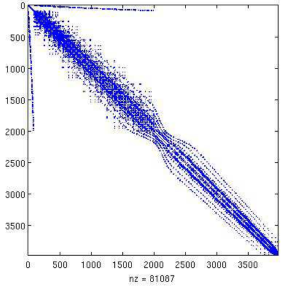
Sparsity pattern of the (left) and of its projection (right) of a walls-roof structure

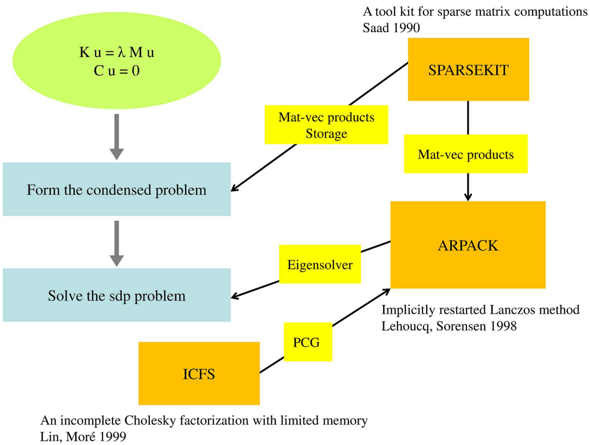


Stiffness matrix K



Projected stiffness matrix $Z^T KZ$

Open-source software to be embedded in NOSA-ITACA



- Performed CPU time competitive with MARC (MSC software).
- In progress: comparison with DACG [Gambolati, Bergamaschi, Pini 1997] and JD [Sleijpen, Van der Vost, Bai 1999]

Case studies

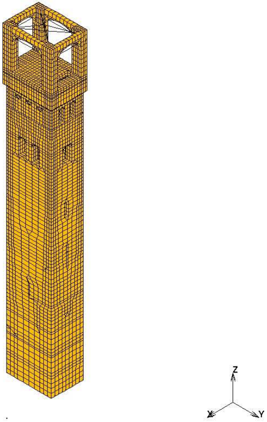
- Torre delle Ore (Lucca): Fixed constraints
- S. Francesco (Lucca): Fixed and Master-Slave constraints of the form

$$u_s = u_m, \;\; s \in I_S, m \in I_M$$

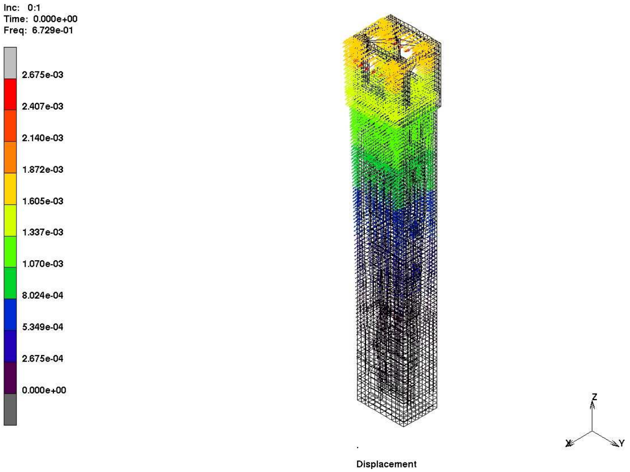
	n	$\lvert I_F \rvert$	$\lvert I_S \rvert$	$\lvert I_M \rvert$
TdO	46164	320	0	0
SF	61881	2662	54	54

	$m = \lvert I_F \rvert + \lvert I_S \rvert$	$N = n - m$	bandwidth
TdO	320	45844	1239
SF	2716	59165	11769

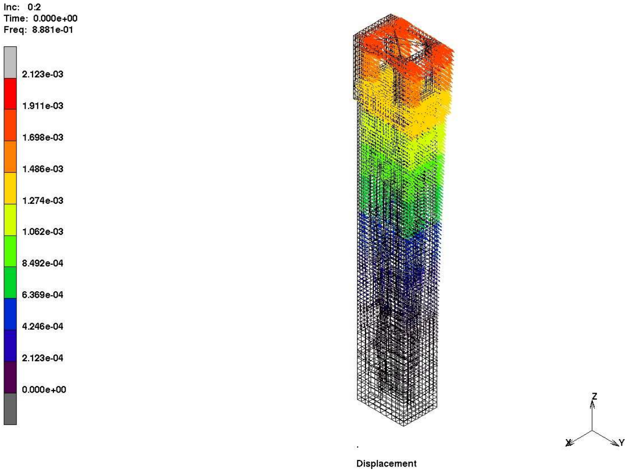
Torre delle Ore



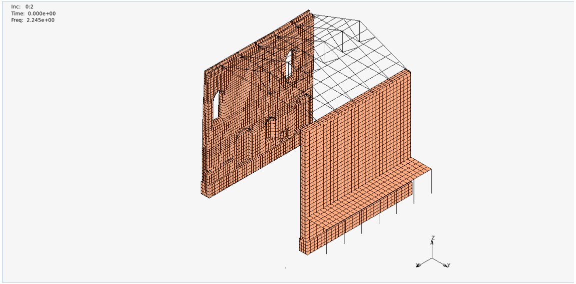
Torre delle Ore: $\nu = 0.67$



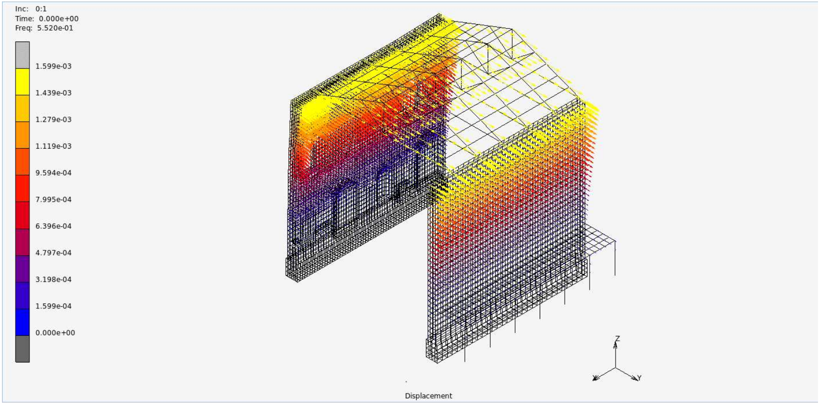
Torre delle Ore: $\nu = 0.88$



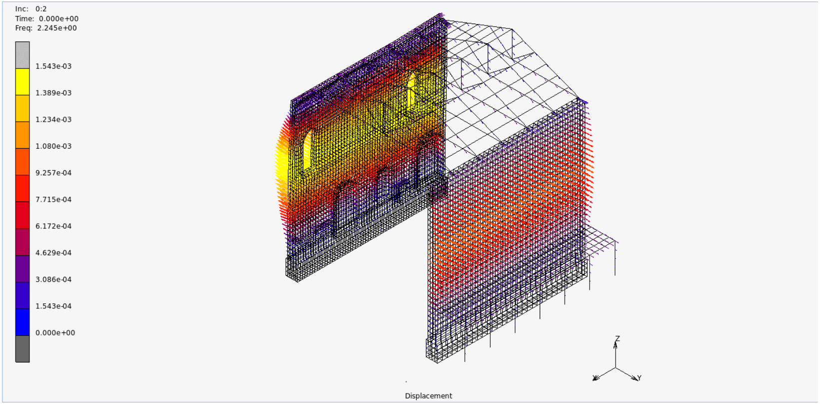
S. Francesco



S. Francesco: $\nu_1 = 0.552$



S. Francesco: $\nu = 2.245$



Thanks for your attention