

Numerical modelling of the static and seismic behaviour of historical buildings: the church of San Francesco in Lucca



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- The NOSA-ITACA code is a finite element solver for nonlinear analyses.
- Masonry is modelled by a nonlinear isotropic elastic material with zero tensile strength and limited compressive strength (masonry-like or no-tension material). [G. Del Piero, *Meccanica* 1989; S. Di Pasquale, *Meccanica* 1992; M. Lucchesi, C. Padovani et al., *Masonry Constructions and Numerical Applications*, Springer 2008].

- Static analyses
- Dynamic analyses
- Thermo-mechanical analyses
- Modal analyses



- Stress fields
- Collapse loads
- Elastic, fracture and crushing strain fields
- Displacement fields
- Time- histories
- Eigenvalues and eigenvectors

- NOSA-ITACA library: beam, truss, shell, 2D, 3D elements (35 elements)

The NOSA-ITACA version for static analyses will be freely downloadable by <http://www.nosaitaca.it/it/software/>

E the infinitesimal strain tensor,
 T the Cauchy stress tensor,
 E^e the elastic part of the strain,
 E^f the fracture strain,
 E^c the crushing strain,
 E, ν the modulus of elasticity and the Poisson's ratio,
 $\sigma_0 < 0$ the masonry maximum compressive stress.

Given E , find E^f, E^c, T such that

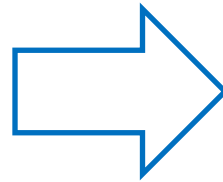
$$E = E^e + E^f + E^c,$$

$$E^f \cdot E^c = 0,$$

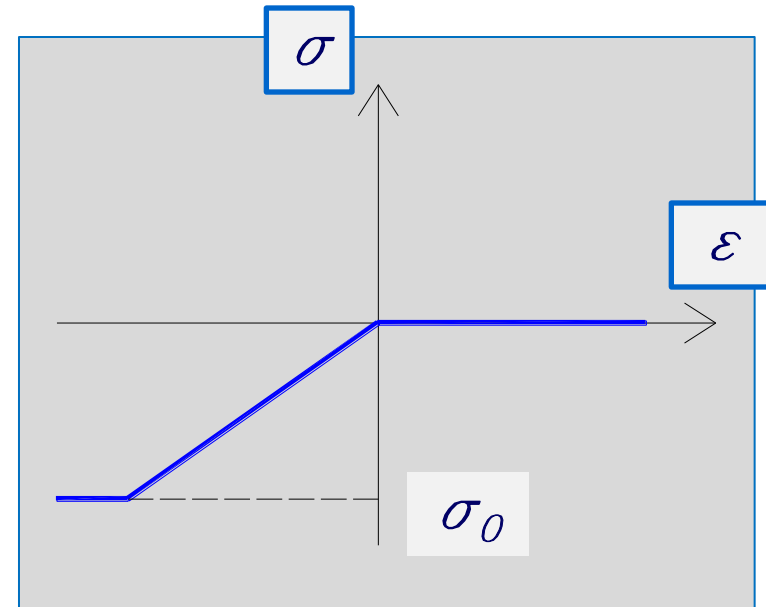
$$T = \frac{E}{1+\nu} \left[E^e + \frac{\nu}{1-2\nu} \text{tr}(E^e) I \right],$$

$$T \cdot E^f = (T - \sigma_0 I) \cdot E^c = 0,$$

$$T, E^c \leq 0, \quad T - \sigma_0 I \geq 0, \quad E^f \geq 0$$

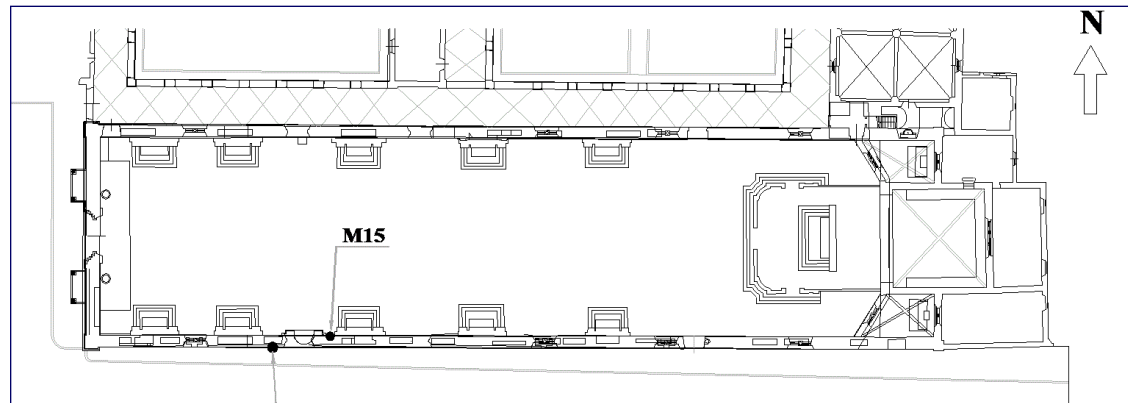
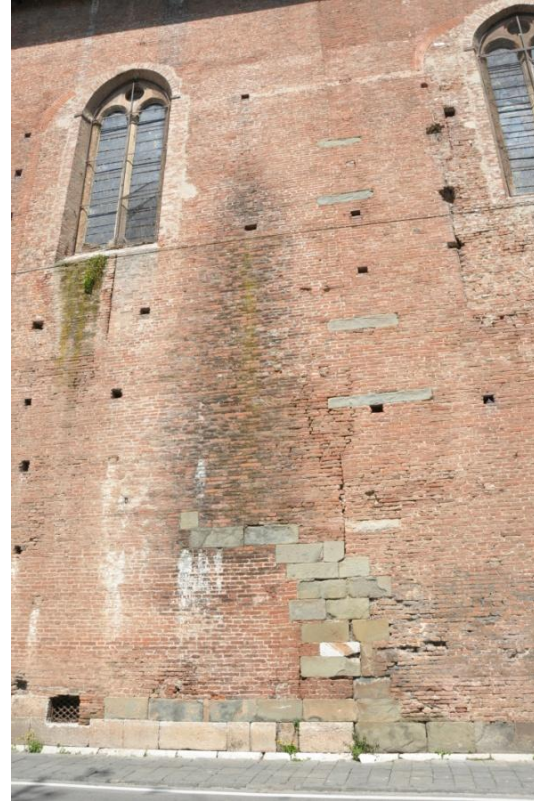


$$T = \hat{T}(E), \quad D_E \hat{T}(E)$$





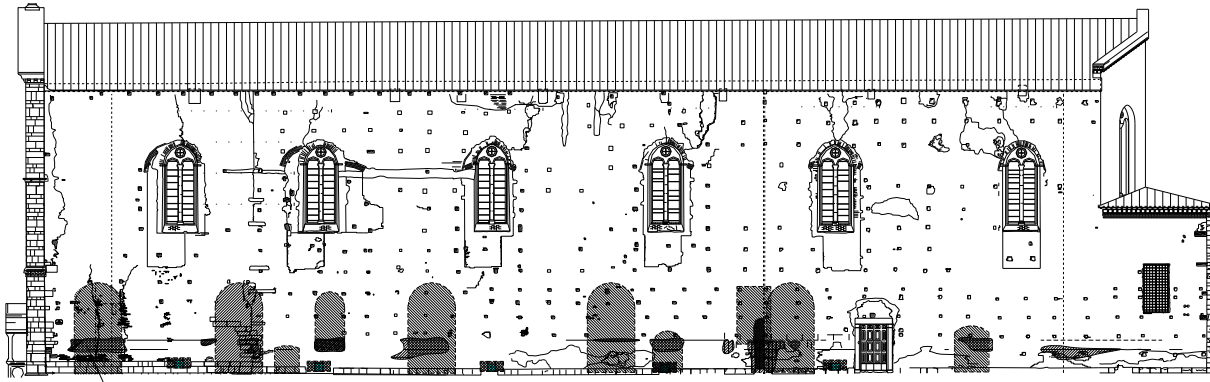
The Church of San Francesco



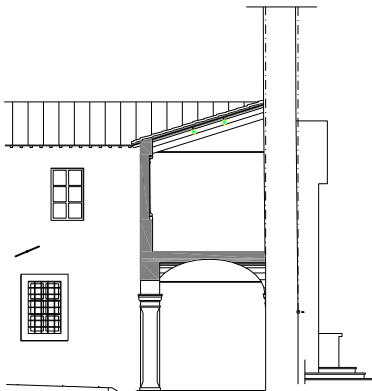
The Church of San Francesco: reinforcement operations



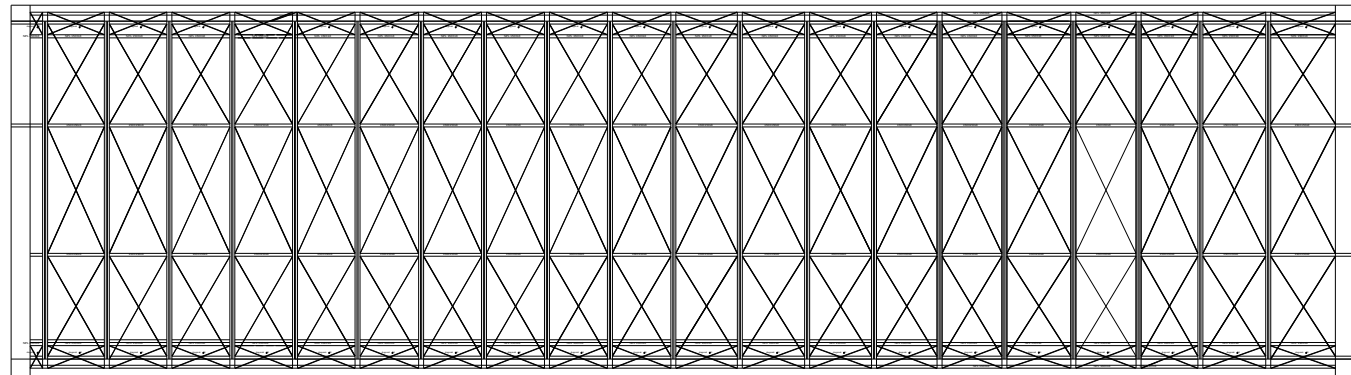
On masonry



On the cloister



On the roof



Evaluation of the seismic vulnerability and the effectiveness of the reinforcement operations

Step 1

- **In situ tests:** geometric survey, flat jacks tests, georadar scans
- **Laboratory tests:** mechanical characterization of materials and soil

Step 2

Numerical model of the existing structure: modal analyses, definition of the horizontal loads, static nonlinear analyses, definition of the collapse loads

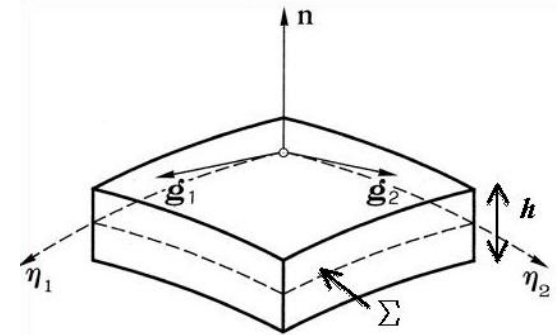
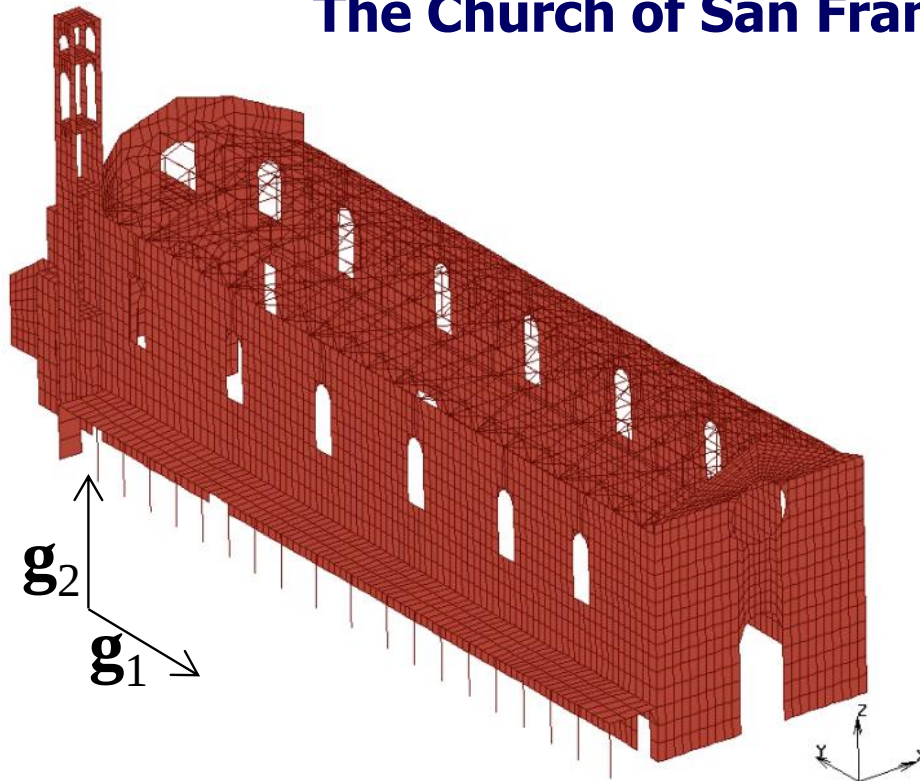
Step 3

Numerical model of the reinforced structure: modal analyses, definition of the horizontal loads, static nonlinear analyses, definition of the collapse loads

Step 4

Existing and reinforced structure: comparison of numerical results

The Church of San Francesco: The finite element mesh



$$\Sigma_\zeta = \{\mathbf{p}' \mid \mathbf{p}' = \mathbf{p} + \zeta \mathbf{n}, \mathbf{p} \in \Sigma, \mathbf{n} = \mathbf{n}(\mathbf{p})\},$$

$$\zeta \in [-h/2, h/2].$$

Stresses

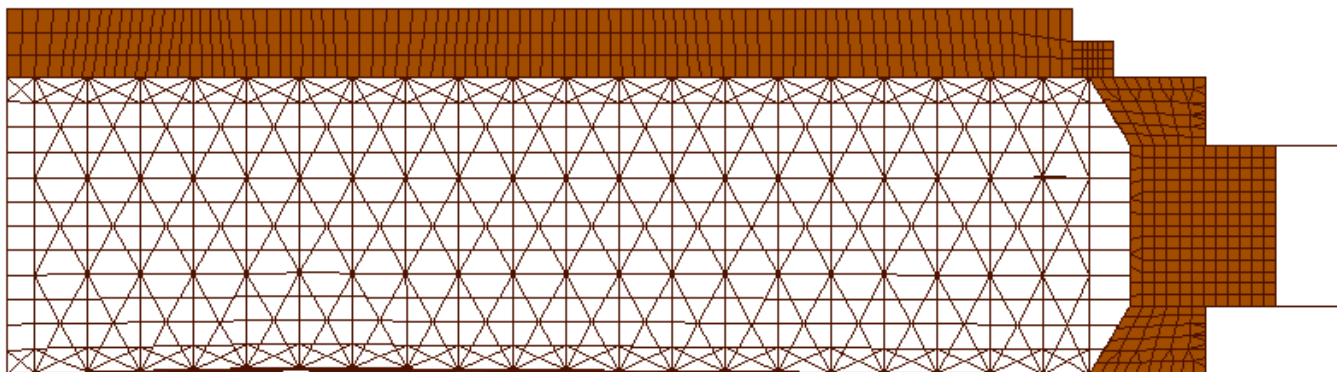
$$T_{11} = \mathbf{g}_1 \cdot \mathbf{T} \mathbf{g}_1, \quad T_{22} = \mathbf{g}_2 \cdot \mathbf{T} \mathbf{g}_2,$$

Normal forces

$$N_{11} = \int_{-h/2}^{h/2} T_{11} d\zeta, \quad N_{22} = \int_{-h/2}^{h/2} T_{22} d\zeta$$

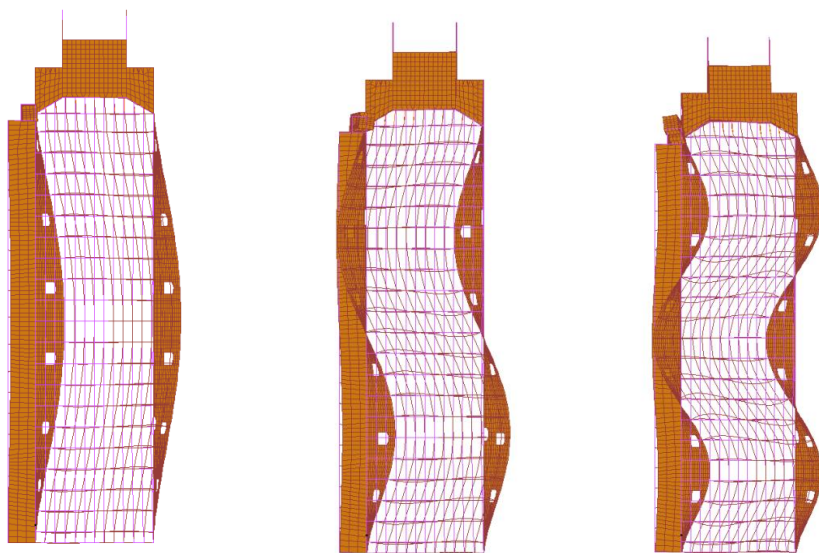
Bending moments

$$M_{11} = \int_{-h/2}^{h/2} T_{11} \zeta d\zeta, \quad M_{22} = \int_{-h/2}^{h/2} T_{22} \zeta d\zeta.$$



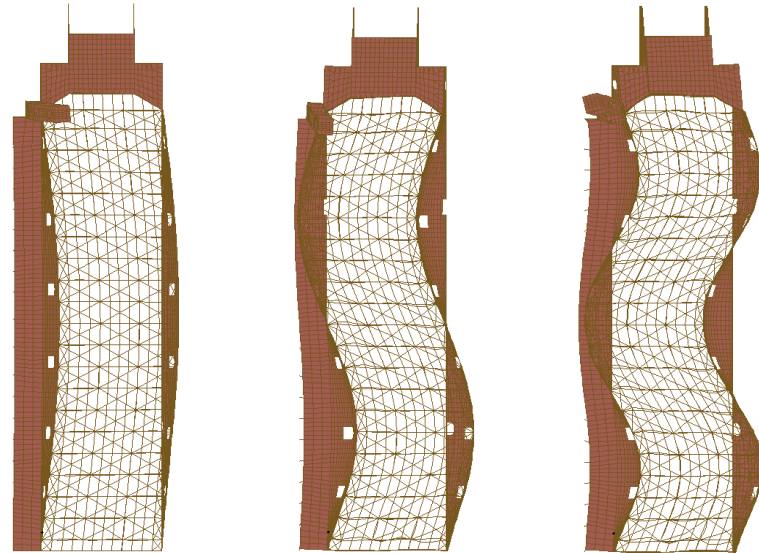
The Church of San Francesco: modal analyses

Without reinforcing

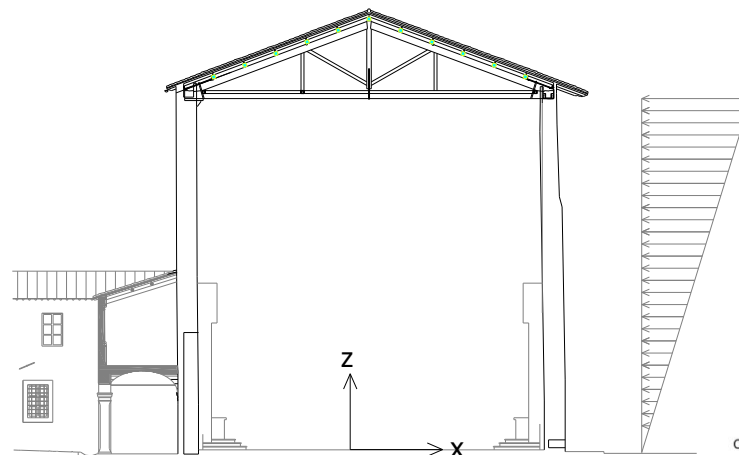
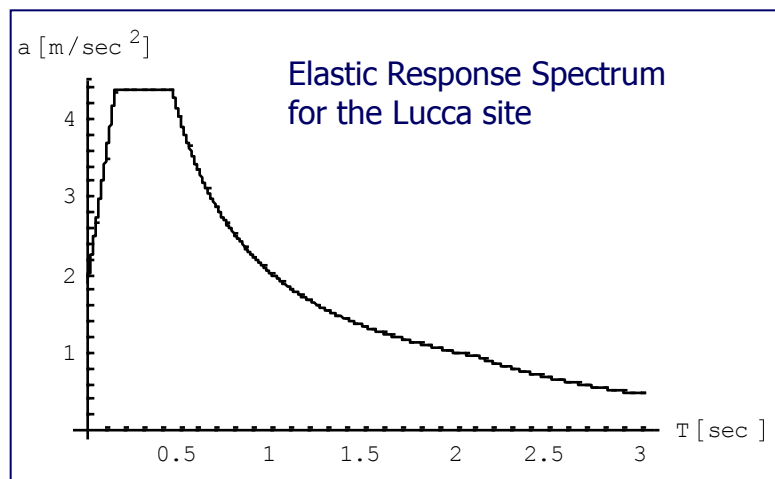


$T_1 = 1,26 \text{ s}$

With reinforcing

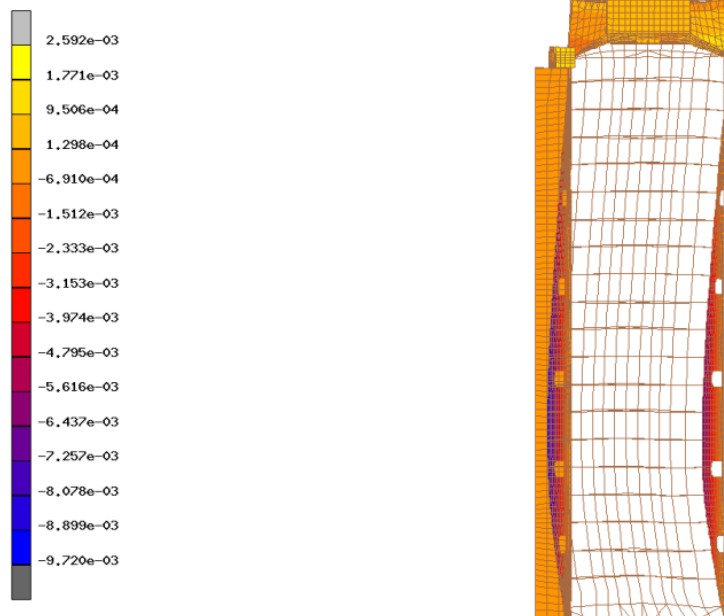


$T_{1r} = 1,04 \text{ s}$



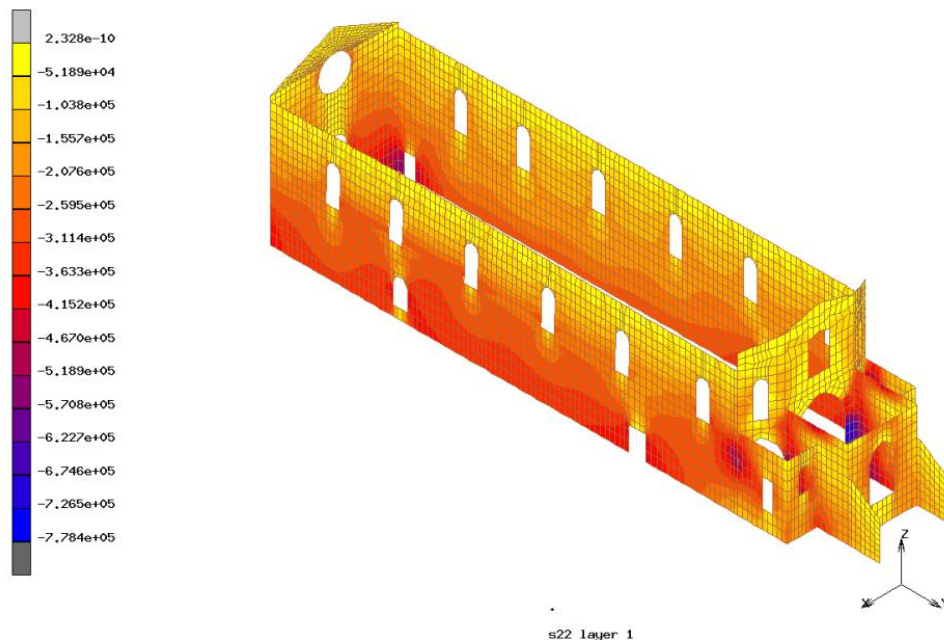
The Church of San Francesco: permanent loads

Inc: 1
Time: 0.000e+00



Displacements u_x

Inc: 1
Time: 0.000e+00



Stresses T_{22}

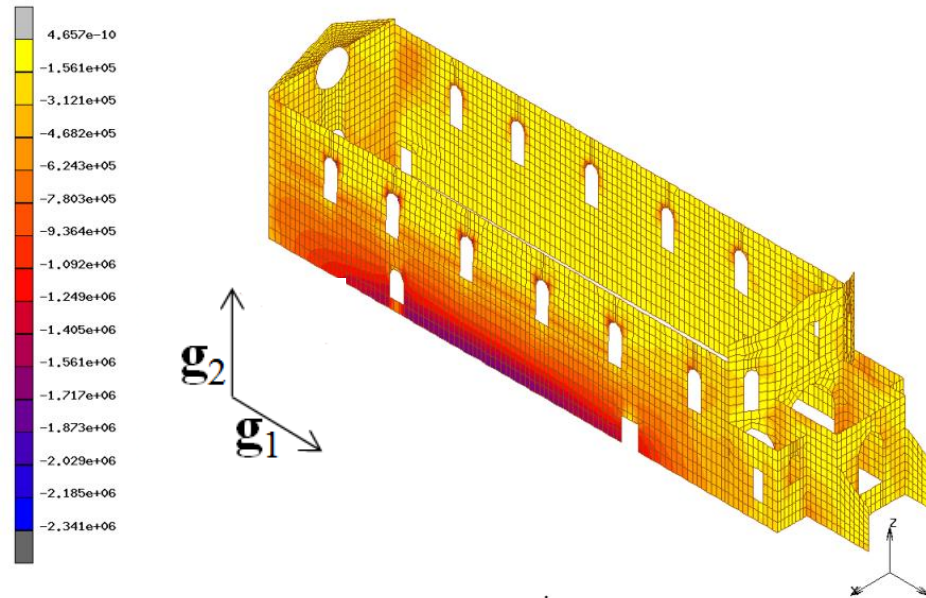
The Church of San Francesco: evaluation of the seismic vulnerability

Stresses T_{22}

SLU

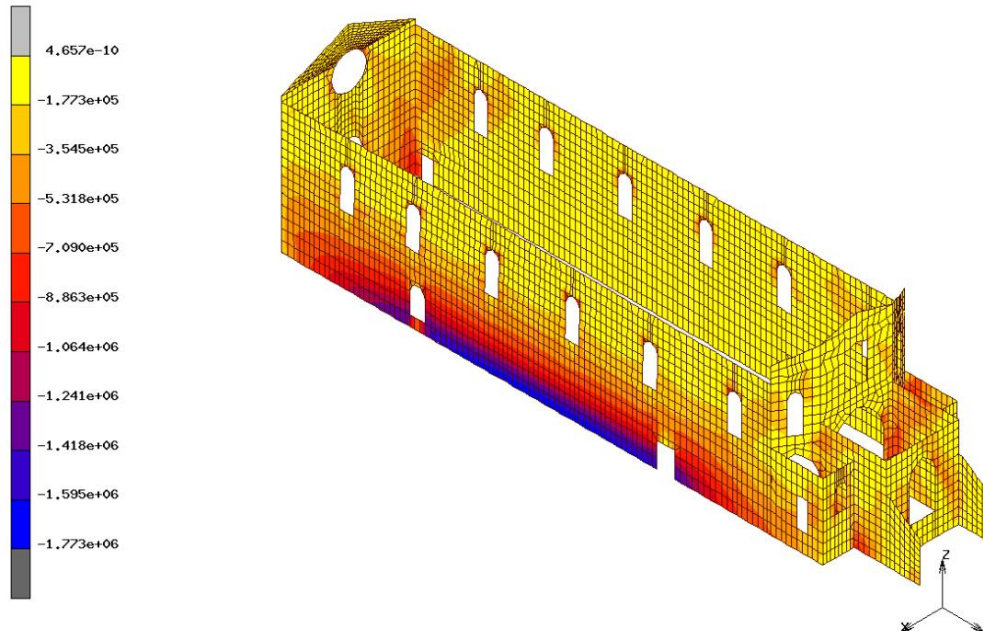
Without reinforcing

$H_u=4,1\%$ Weigth

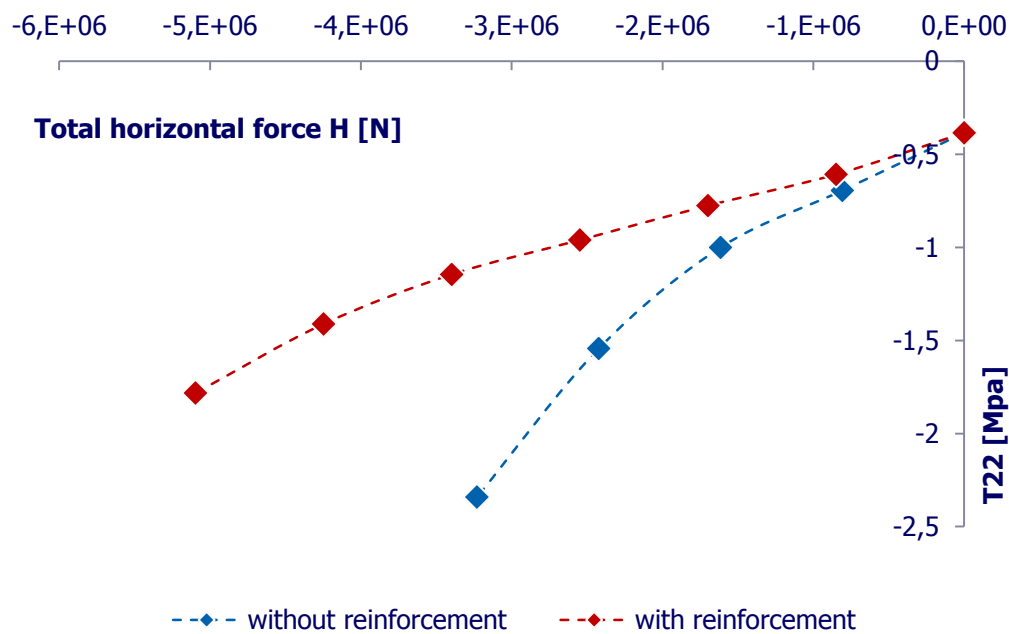


With reinforcing

$H_u=6,5\%$ Weigth



The Church of San Francesco: evolution of the maximum values of normal stresses



The Church of San Francesco: evaluation of the seismic vulnerability

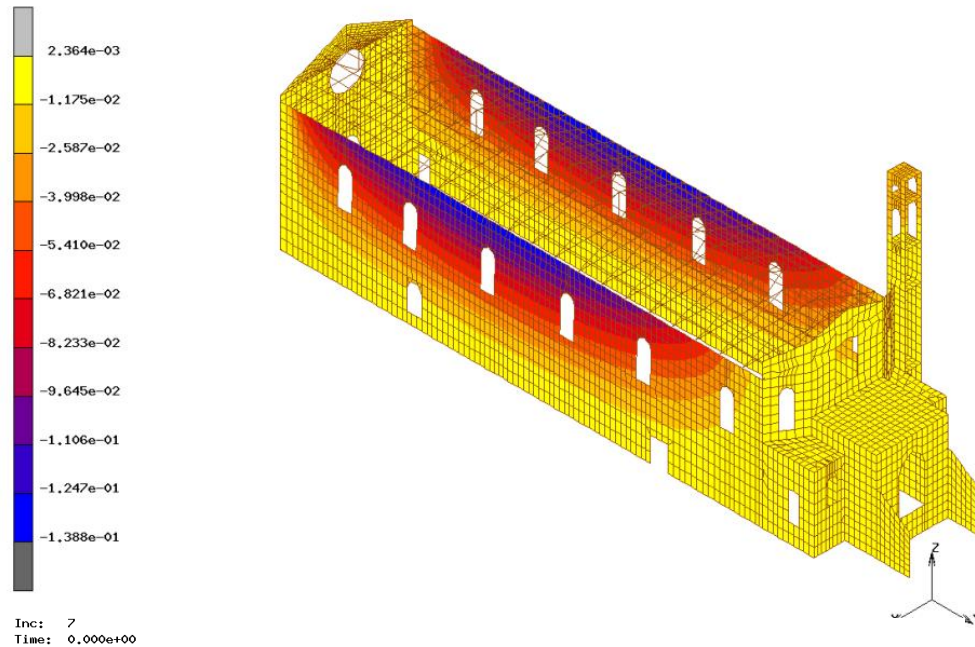
Displacements u_x

SLU

Without reinforcing

$H_u = 4,1\%$ Weight

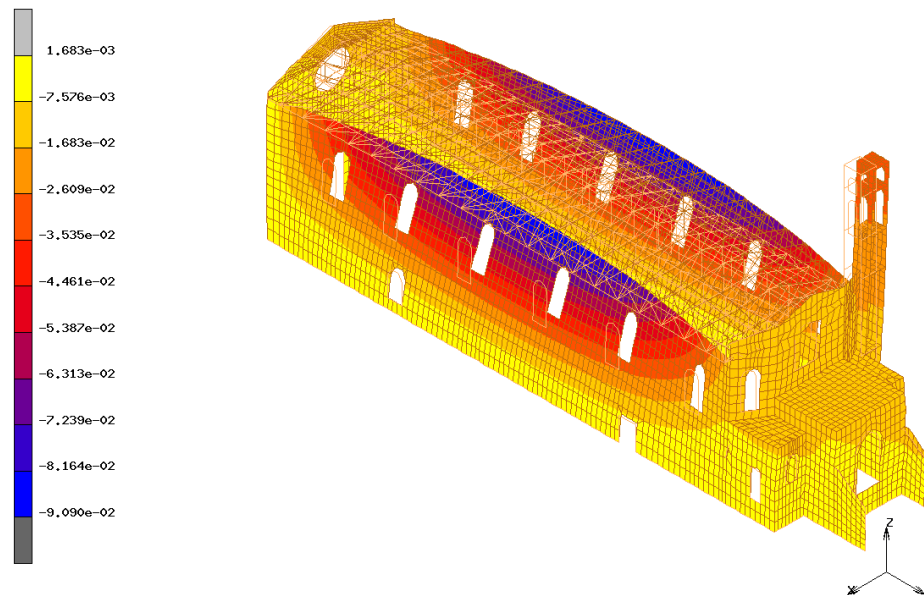
$U_{x\max}/h = 0,0078$



With reinforcing

$H_u = 6,5\%$ Weight

$U_{x\max}/h = 0,005$



The Church of San Francesco: evaluation of the seismic vulnerability

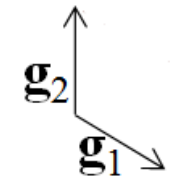
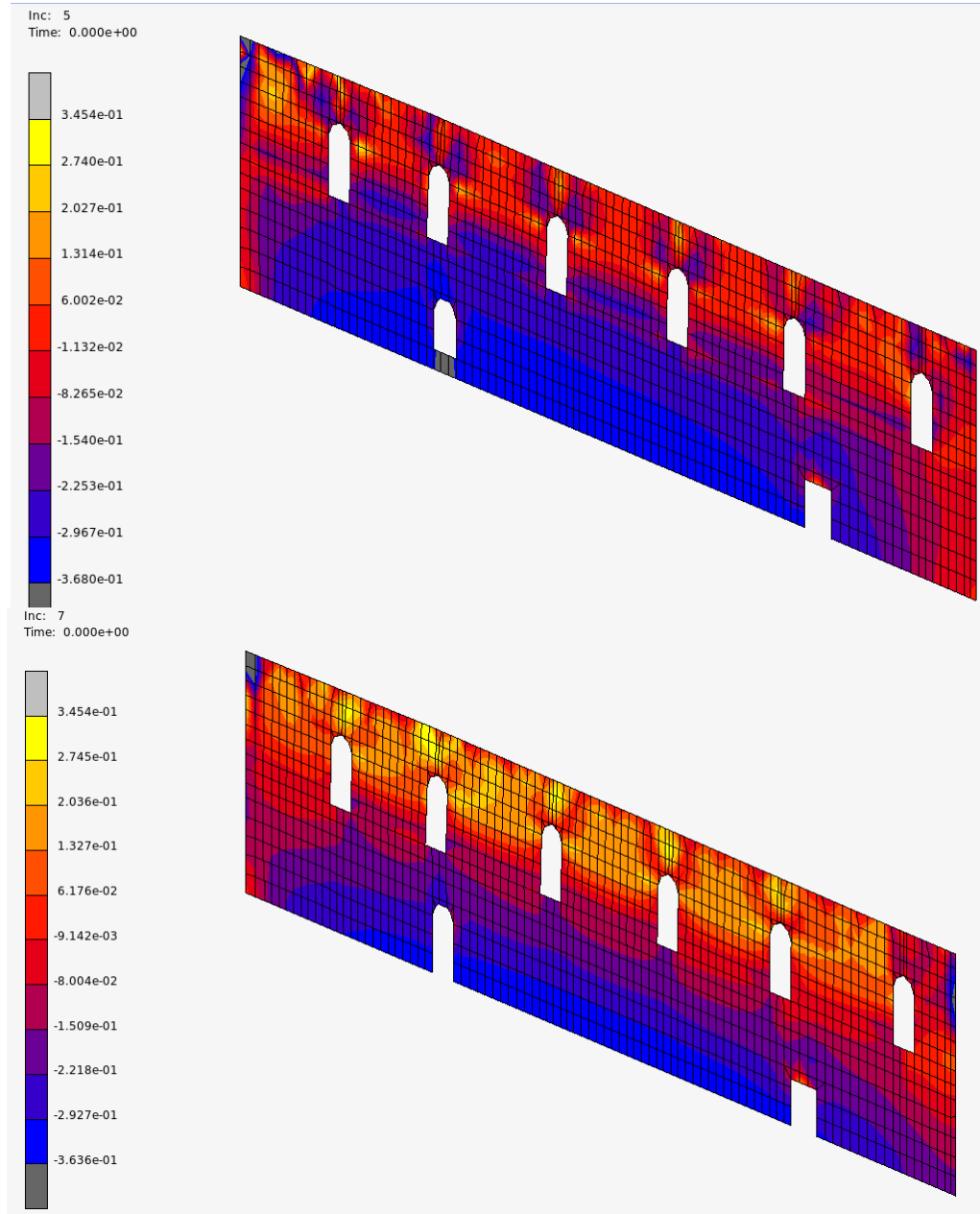
Eccentricities e_{22}

SLU

Without reinforcing

$$e_{22} = M_{22}/N_{22}$$

With reinforcing

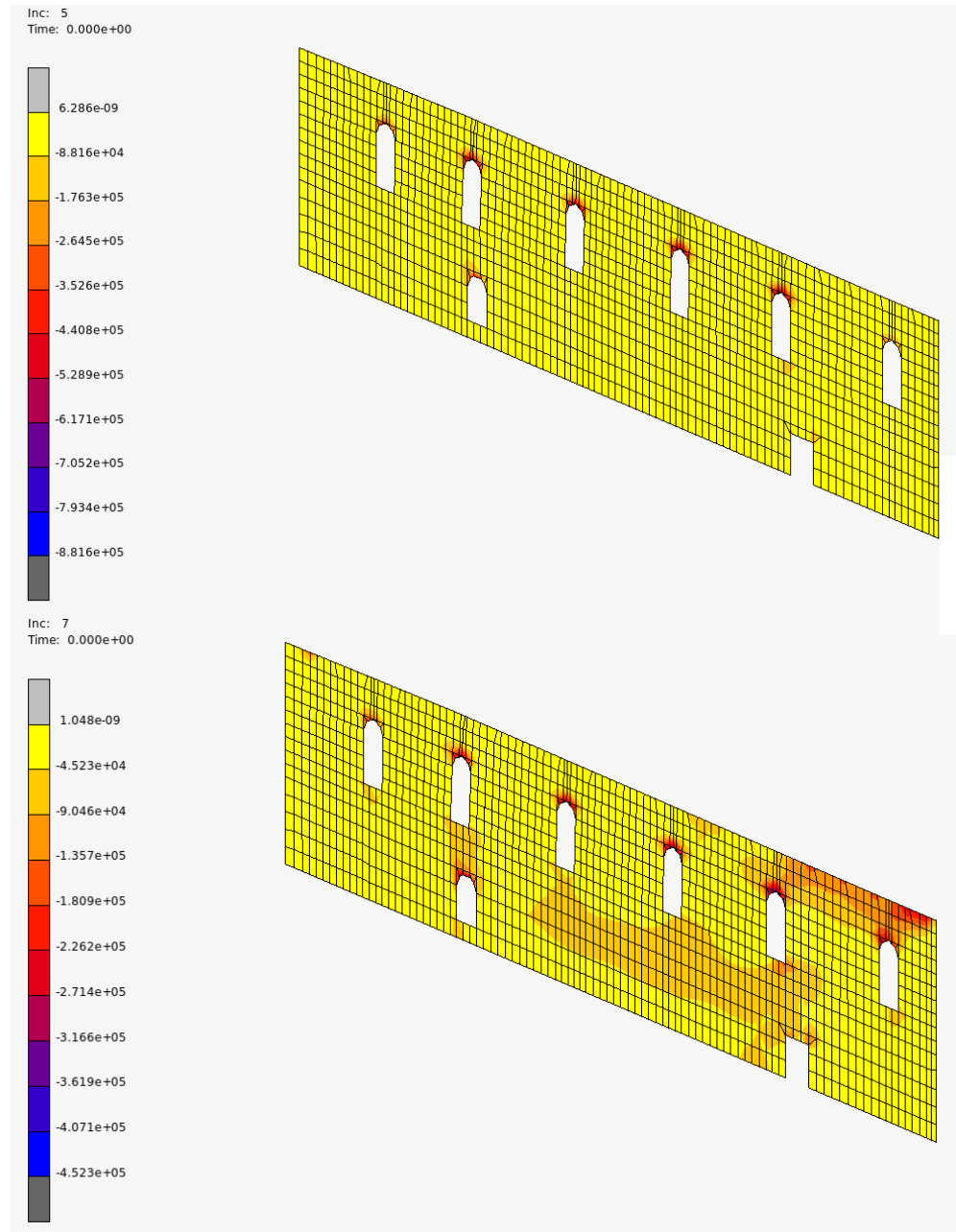


Stresses T_{11}

SLU

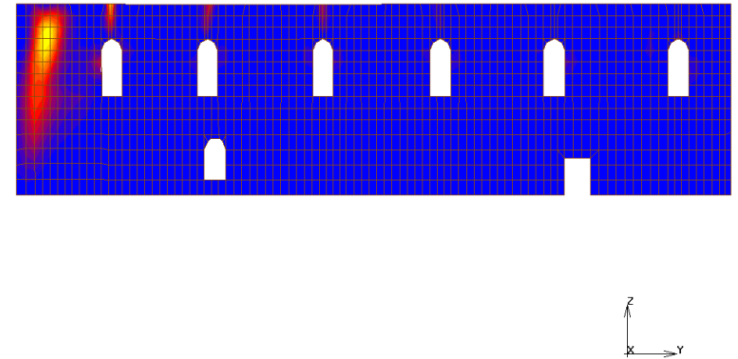
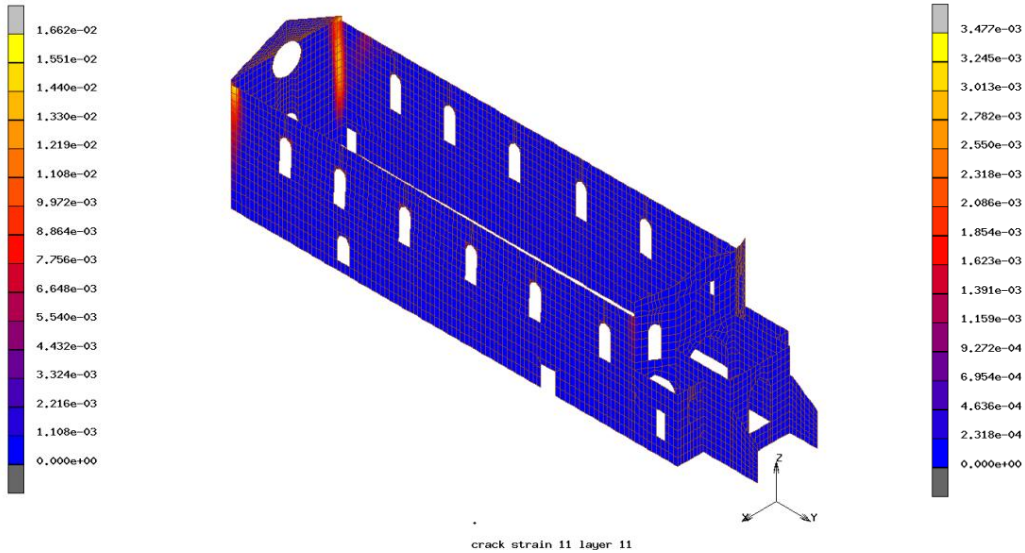
Without reinforcing

With reinforcing



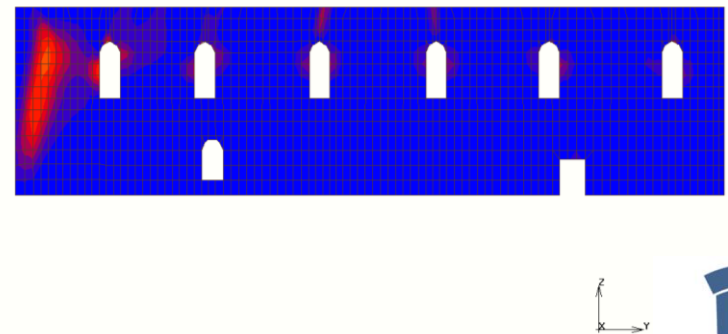
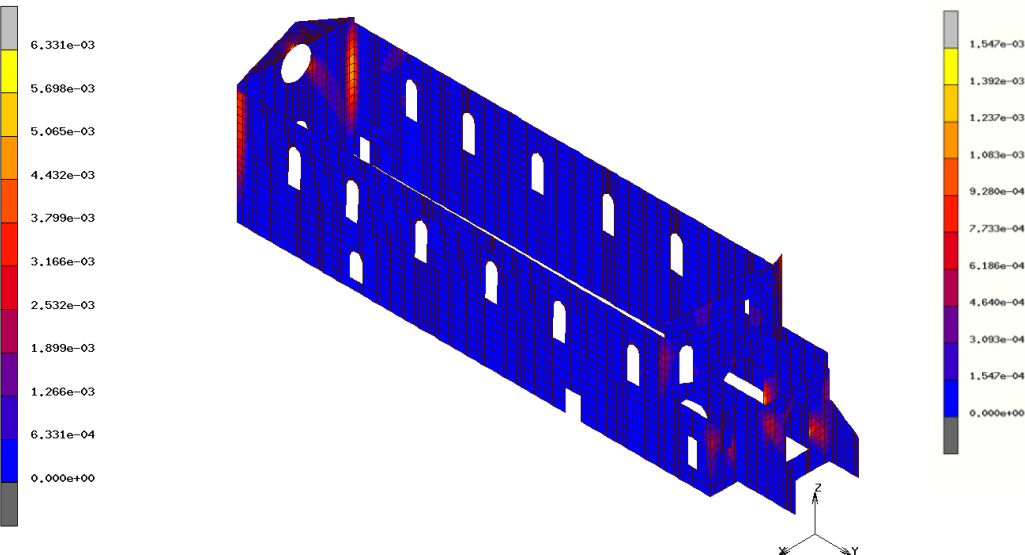
Without reinforcing

Inc: 5
Time: 0,000e+00



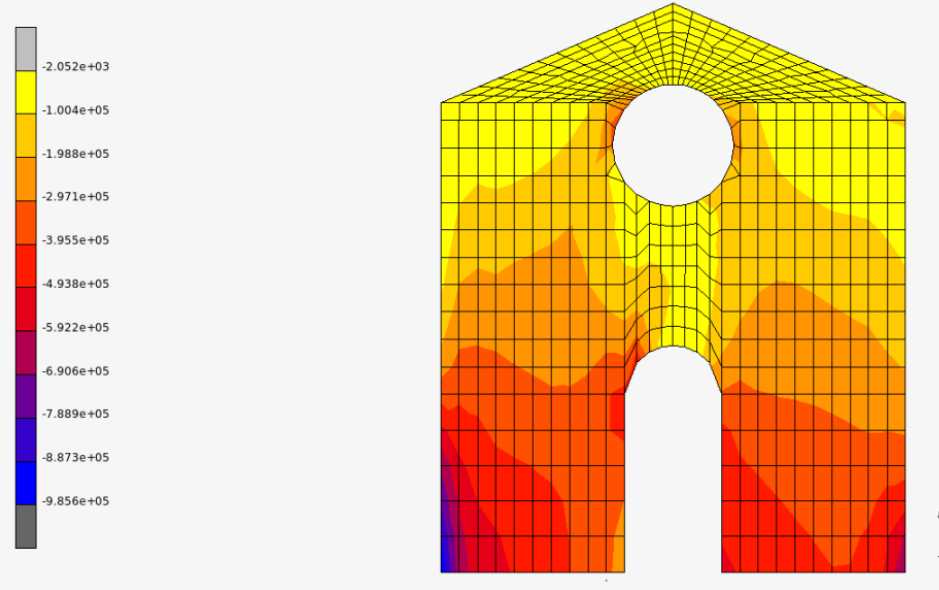
With reinforcing

Fracture strains E^f_{11}

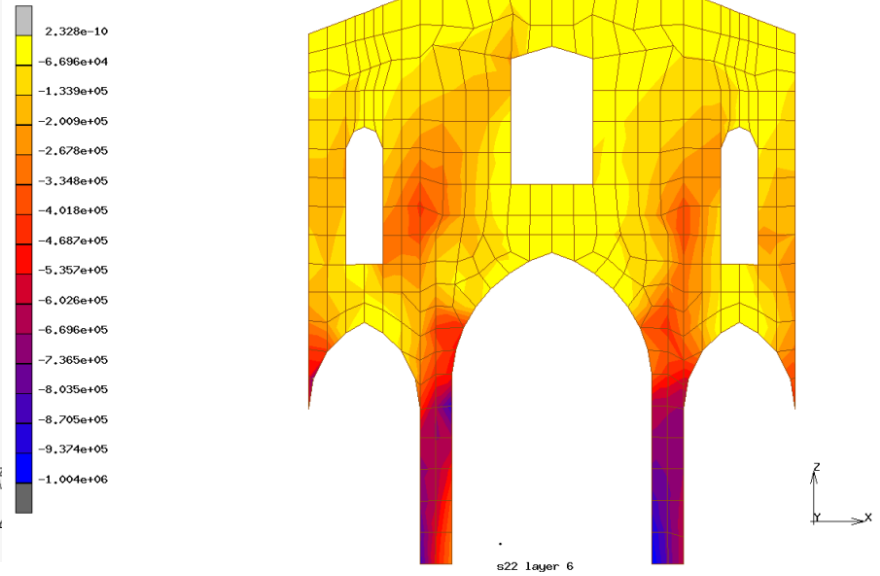


Without reinforcing

Inc: 5
Time: 0.000e+00

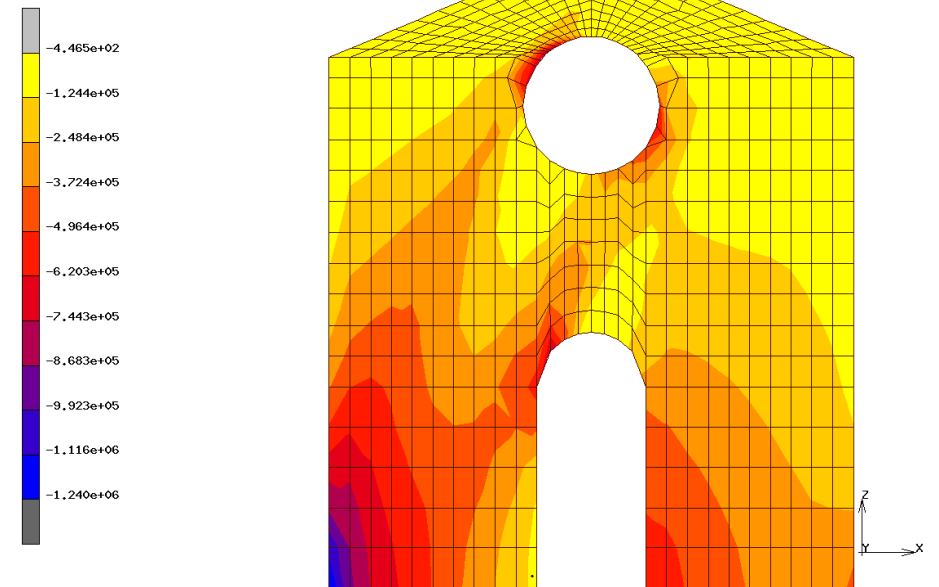


Inc: 5
Time: 0.000e+00

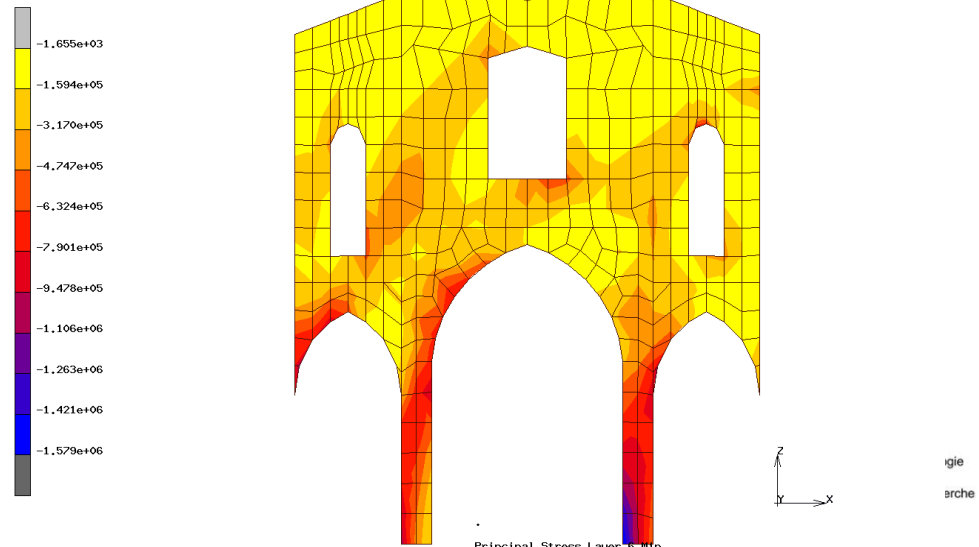


With reinforcing

Inc: 7
Time: 0.000e+00



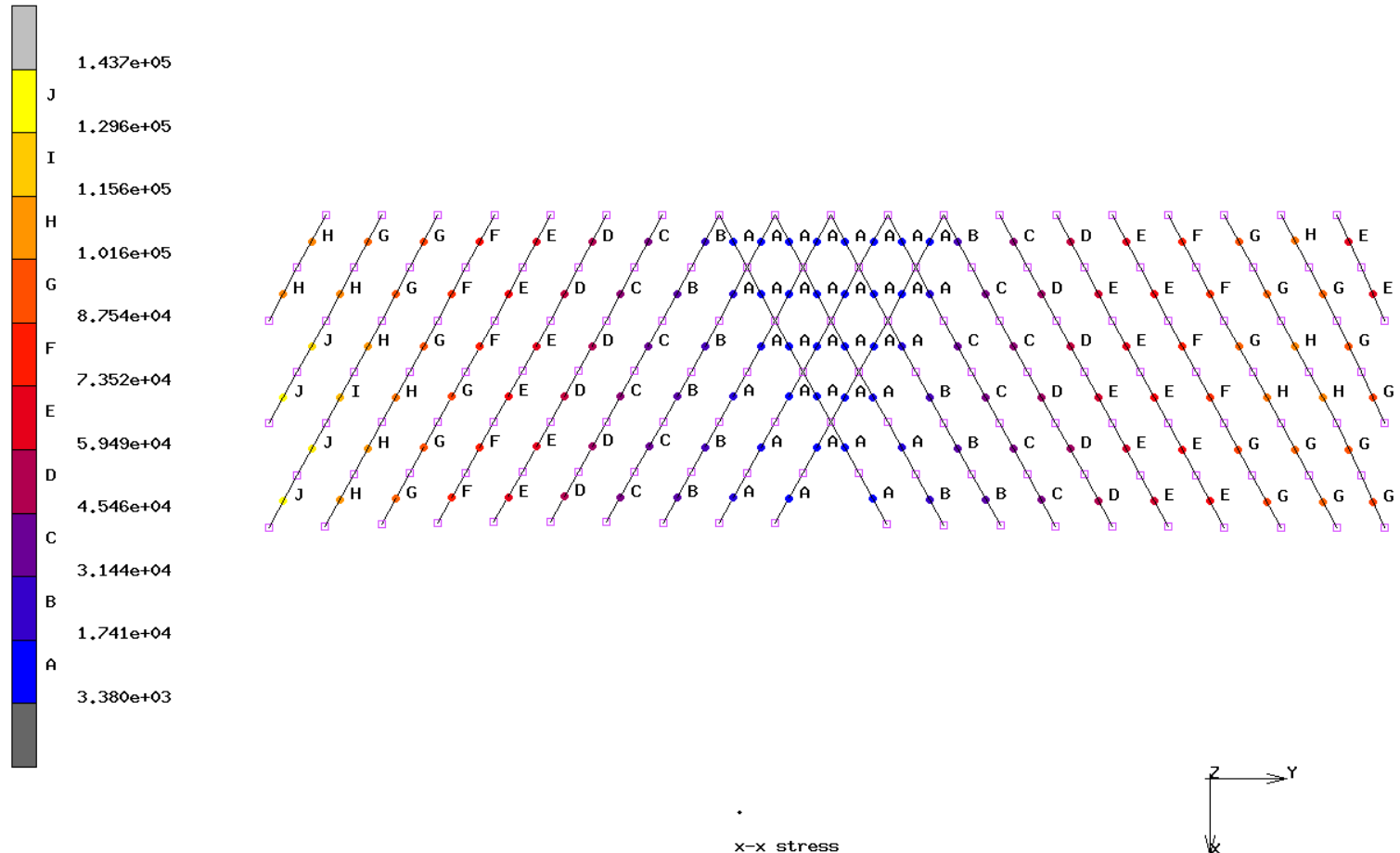
Inc: 7
Time: 0.000e+00



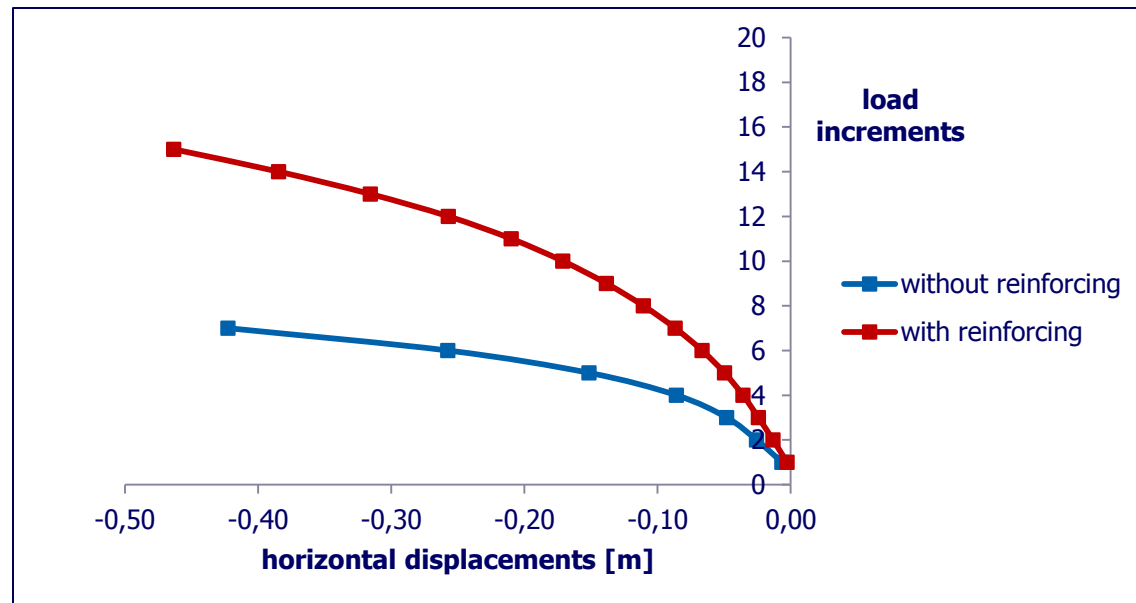
Stresses T_{22}

Normal force in the windbracing at the ultimate limit state

Inc: 7
Time: 0.000e+00



The Church of San Francesco: evaluation of the seismic vulnerability



Without reinforcing

$$f_a = \frac{a_{SLU}}{a_{475}} = \frac{0.75}{1.86} = 0.4$$

$$I_s = \frac{50}{475} = 0.1$$

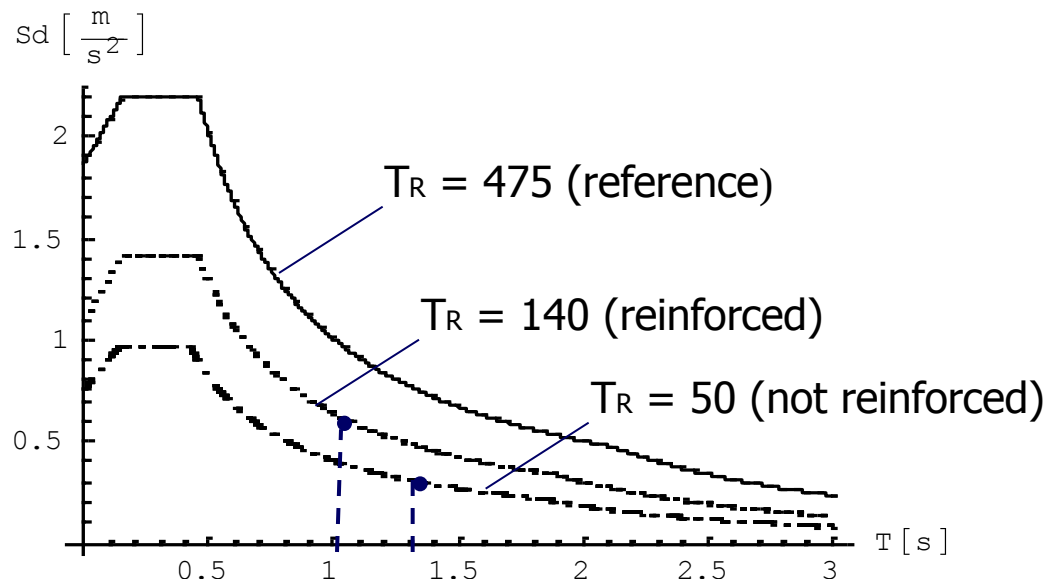
$$\frac{H_{SLU}}{W} = 0.041$$

With reinforcing

$$f_a = \frac{a_{SLU}}{a_{475}} = \frac{1.16}{1.86} = 0.6$$

$$I_s = \frac{140}{475} = 0.3$$

$$\frac{H_{SLU}}{W} = 0.065$$



Conclusions

- The NOSA - ITACA code is a finite element code for static and dynamic nonlinear analyses of masonry structures. The version for static analyses will be freely downloadable by the end of the year.
- Masonry is modelled by means of a masonry-like constitutive equation with zero tensile strength and finite or infinite compressive strength.
- A case study has been presented in which the seismic vulnerability of the church of San Francesco in Lucca is assessed by means of a nonlinear static analysis conducted via the NOSA–ITACA code.

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