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Nonlinear Analyses on the medieval "Ponte del Diavolo" in Borgo a Mozzano (Italy)

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Finite Element Code **NOSA-ITACA**[1]

- **NOSA-ITACA** is a freeware/open source software package developed by ISTI together with DICEA. It is part of the project *'Tools for modeling and assessing the structural behavior of ancient constructions'* supported by the Region of Tuscany.
- **NOSA-ITACA** is a finite element code that combines NOSA[2] with an open source graphic platform called SALOME. It is used to study the static and dynamic behavior of masonry constructions.
- **Masonry** is described as a nonlinear elastic material with zero tensile strength and infinite or bounded compressive strength.

- Static analysis
- Dynamic analysis

→

- Stress field
- Collapse load
- Elastic, cracking and crushing strain
- Displacement fields
- Time-histories

- NOSA-ITACA library: beam, shell, 2D and 3D elements (35 elements).

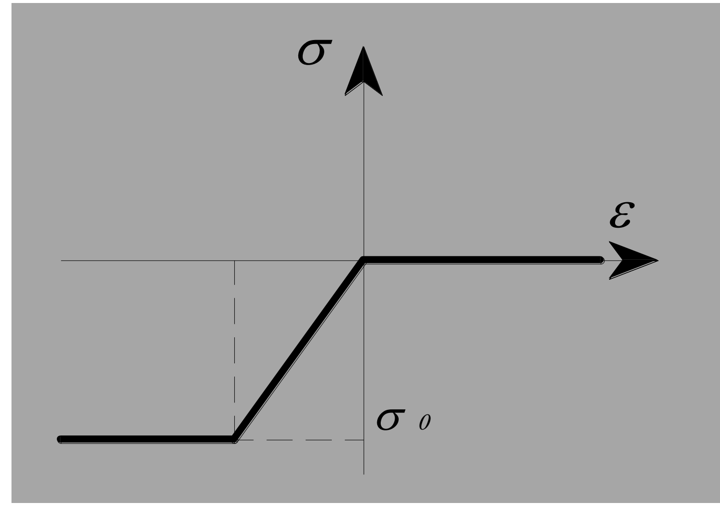
[1] www.nosaitaca.it

[2] M. Lucchesi, C. Padovani, G. Pasquinelli, N. Zani, "Masonry constructions: mechanical models and numerical applications", Series: Lecture Notes in Applied and Computational Mechanics, Vol. 39, Berlin Heidelberg, Springer-Verlag, 2008

Finite element code **NOSA-ITACA** [1]

The constitutive equation of ***masonry-like*** materials

$\boldsymbol{E}$	infinitesimal strain tensor,
$\boldsymbol{T}$	Cauchy stress tensor,
$\boldsymbol{E}^e$	elastic part of the strain,
$\boldsymbol{E}^f$	fracture part of the strain,
$\boldsymbol{E}^c$	crushing part of the strain,
E, ν	Young 's modulus and Poisson's modulus,
$\sigma_0 < 0$	maximum resistance to compression.



Given $\boldsymbol{E}$, it is possible to obtain $\boldsymbol{E}^f$, $\boldsymbol{E}^c$, $\boldsymbol{T}$ such that :

$$\boldsymbol{E} = \boldsymbol{E}^e + \boldsymbol{E}^f + \boldsymbol{E}^c$$

$$\boldsymbol{E}^f \cdot \boldsymbol{E}^C = 0$$

$$\boldsymbol{T} = \frac{\mathbf{E}}{1+\boldsymbol{\nu}}\left[\boldsymbol{E}^e + \frac{\boldsymbol{\nu}}{1+\boldsymbol{\nu}} \boldsymbol{tr}(\boldsymbol{E}^e)\boldsymbol{I}\right]$$

$$\boldsymbol{T} \cdot \boldsymbol{E}^f = (\boldsymbol{T} + \sigma_0 \boldsymbol{I}) \cdot \boldsymbol{E}^c = 0$$

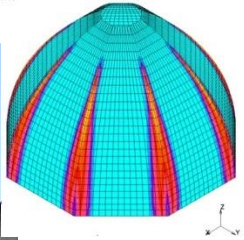
$$\boldsymbol{T}, \boldsymbol{E}^c \in Sym^-, \boldsymbol{E}^f \in Sym^+, \ (\boldsymbol{T} - \sigma_0 \boldsymbol{I}) \in Sym^+$$

$$\Rightarrow \quad \boldsymbol{T} = \widehat{\boldsymbol{T}}(\boldsymbol{E}), \qquad \mathbf{D_E}\widehat{\boldsymbol{T}}(\boldsymbol{E})$$

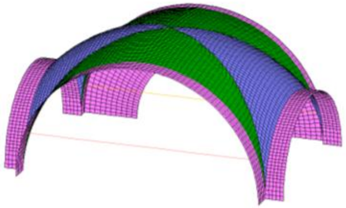
[1] www.nosaitaca.it

Finite Element Code **NOSA-ITACA** : some applications

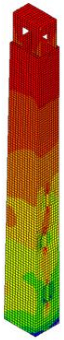
- 1995 Battistero del Duomo, Volterra
- 1996 Arsenale Mediceo, Pisa
- 1998 Teatro Goldoni, Livorno
- 1998 Chiesa Madre di S. Nicolò, Noto
- 2004 Chiesa di Santa Maria Maddalena, Morano Calabro (CS)

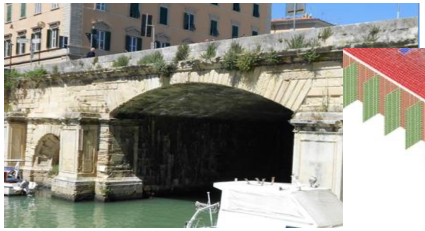
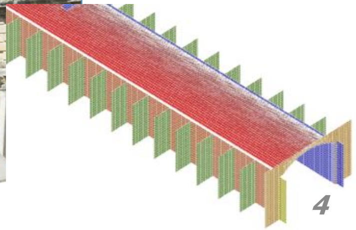
- 2005 Chiesa di San Ponziano, Lucca

- 2008 Chiesa Abbaziale di Santa Maria della Roccella, Scolacium
- 2008 Torre “Rognosa” , San Gimignano

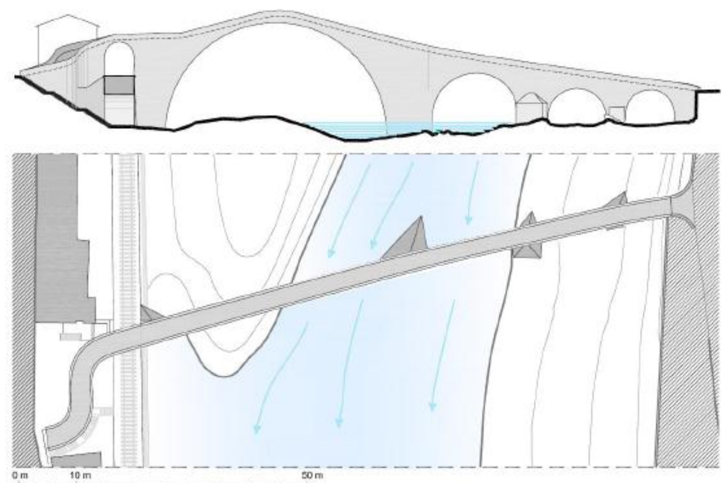



- 2010 Torre “delle Ore”, Lucca
- 2012 Chiesa di San Francesco, Lucca
- 2013 Dome of Cathedral of Massa Marittima(?), Grosseto
- 2013 The “Voltone”, Livorno

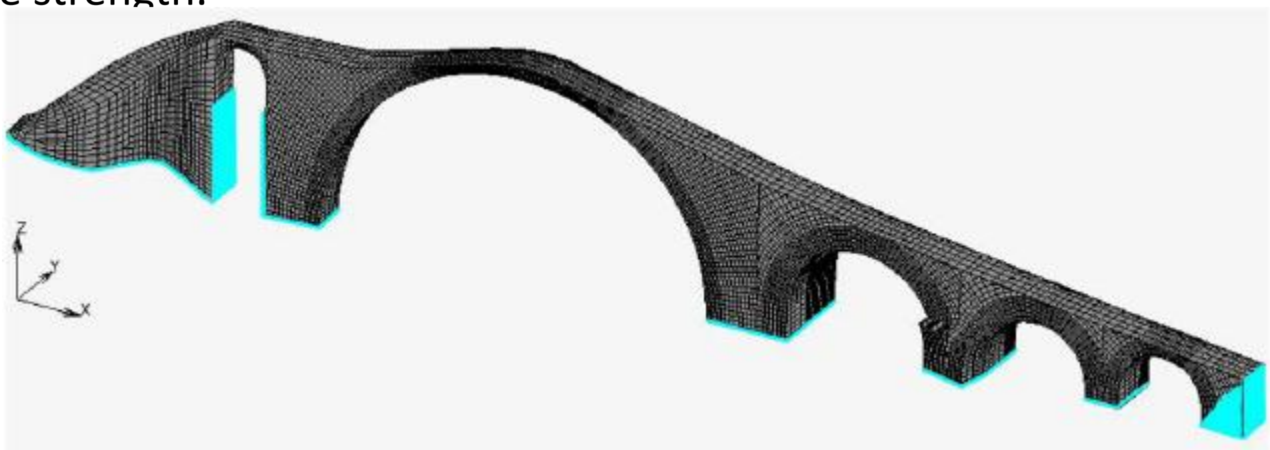
Case study : The Maddalena Bridge

The Maddalena bridge in Borgo a Mozzano, also known as the ***Devil's Bridge***, is one of Italy's most intriguing medieval bridges. This sandstone and limestone masonry construction crosses the Serchio River for a total length of about one hundred meters. As it follows the positions of rocky outcrops, its path deviates from the direction perpendicular to the river by more than fifteen degrees. The bridge is made up of four semicircular arches having clear spans of 38 m, 14.5 m, 10.5 m and 8.5 m, and a transverse section between 3.5 m and 3.7 m wide.



The bridge under its self-weight

Two finite element models have been defined using eight-node isoparametric brick elements (model 1) and four-node thick shell elements (model 2). For all analyses the bridge's self-weight alone was considered; its constituent materials are modeled as a homogeneous masonry-like material with zero tensile strength and bounded compressive strength.

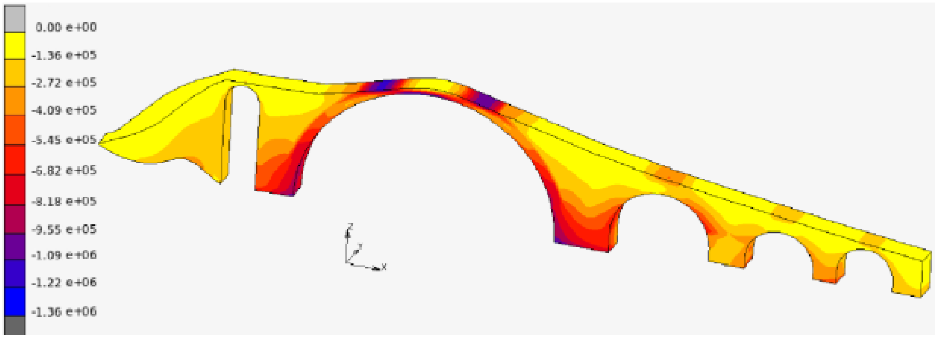


Model 1: the brick elements model (30156 elements; 38626 nodes). Restrained nodes are highlighted

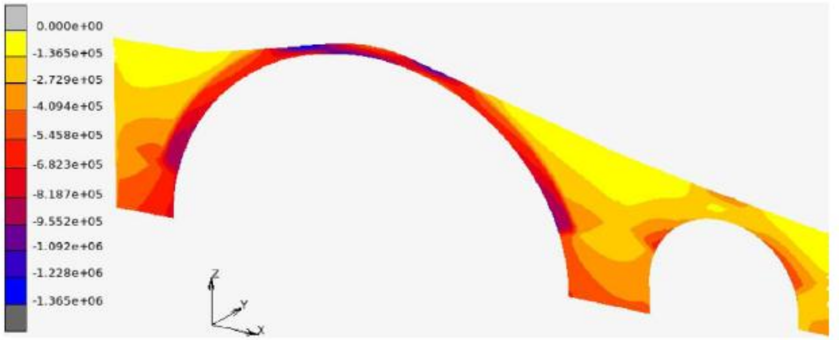
Masonry type	Elastic modulus, E [MPa]	Unit weight, γ [kN/m^3]	Poisson ratio, ν
Sandstone masonry (arch barrel)	8000	24	0.2
Limestone (remaining structure)	3000	24	0.2

Mechanical properties of the material used in the model

The bridge under its self-weight

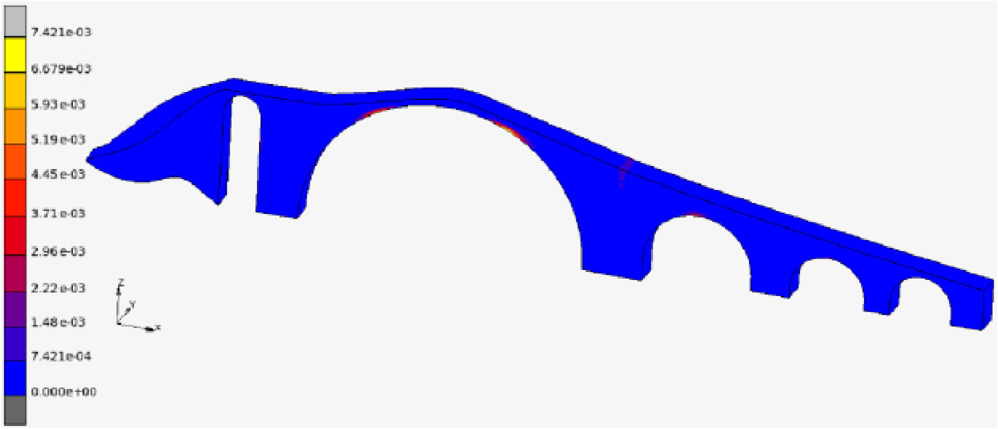


Model 1: minimum principal values of the stress tensor [N/m^2]



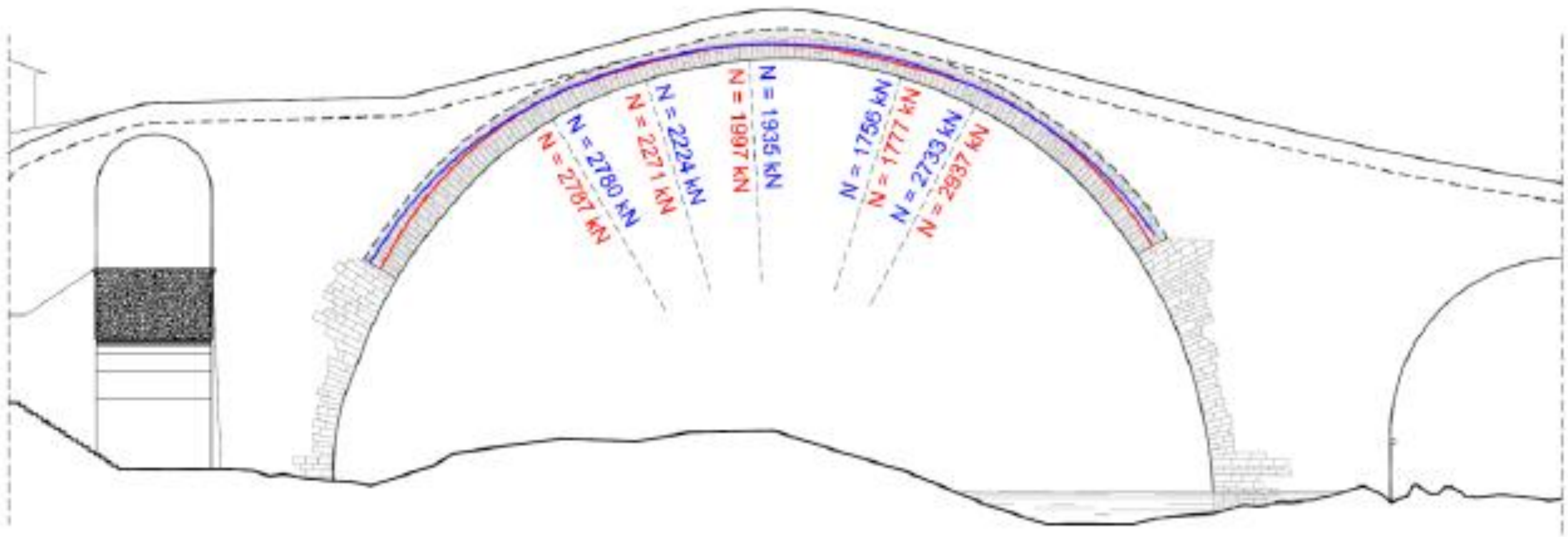
Model 2: minimum principal values of the stress tensor [N/m^2]

- Equilibrium convergence reached with zero material tensile strength
- Maximum compressive stress at base has a value of 1.3 Mpa



Model 1: maximum principal f racture strains

The bridge under its self-weight



Line of thrust on the main arch for different values of Young's modulus of the arch ring material:
E=8000 MPa (blu line), E=3000 Mpa (red line)

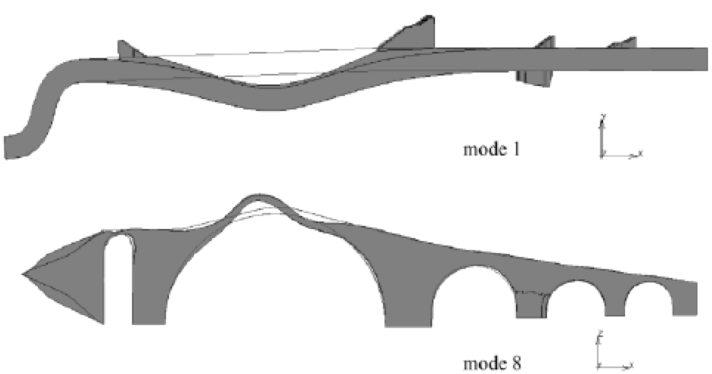
Sensitivity analysis was also performed by changing the material characteristics of the main arch barrel. The results are plotted in terms of line of thrust; the thrust intensity values on the transverse section of the arch ring are reported in the radial direction.

Dynamic Analyses: Modal analyses

	Brick model (Model 1)				Shell model (Model 2)			
n°	ν [Hz]	mass x	mass y	mass z	ν [Hz]	mass x	mass y	mass z
1	2.18	0.002	**0.19**	5.9E-05	2.19	1.7E-04	**0.22**	2.6E-07
2	3.86	4.9E-04	2.5E-02	3.3E-04	4.09	4.2E-04	0.031	1.7E-06
3	4.94	**0.19**	2.8E-02	5.4E-04	5.12	**0.21**	1.8E-05	1.3E-04
4	5.21	2.4E-02	**0.16**	2.4E-06	5.57	4.6E-05	**0.24**	3.5E-07
5	6.56	0.009	5.1E-04	1.6E-03	6.68	0.022	3.3E-04	4E-05
6	6.84	3.6E-03	4.4E-04	9.4E-04	7.21	4E-04	3.3E-03	3.7E-06
7	7.86	2.7E-02	6.9E-02	1.5E-05	8.25	0.0127	0.085	1.1E-06
8	9.03	**0.22**	8.9E-03	1.4E-04	9.11	**0.23**	0.005	2E-04
9	9.71	8.9E-04	5.6E-05	2.4E-05	9.86	1.2E-05	5E-06	**0.13**
10	9.75	1.6E-05	2.0E-04	**0.12**	10.1	4E-05	0.002	7E-05

A preliminary modal analysis was performed in order to check the fundamental frequencies and modal shape of the bridge and their participing masses. Note that:

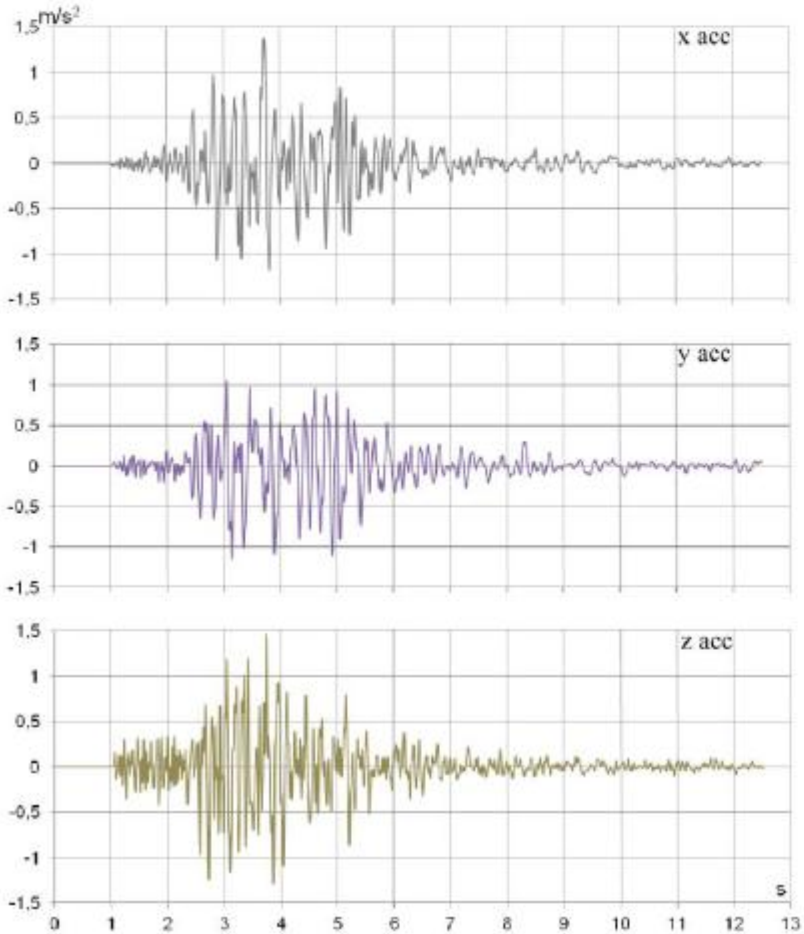
- the first eight vibration modes are quite similar;
- the participating mass associated with the fourth mode are different;
- the 10° mode of model 1 corresponds to the 9° in model 2.



Model 1 : Two examples of modes shape

The fully three-dimensional model is able to capture more complex out of plane and torsional vibration modes. However, model 2 accounts for the most important features of the bridge's dynamic behavior.

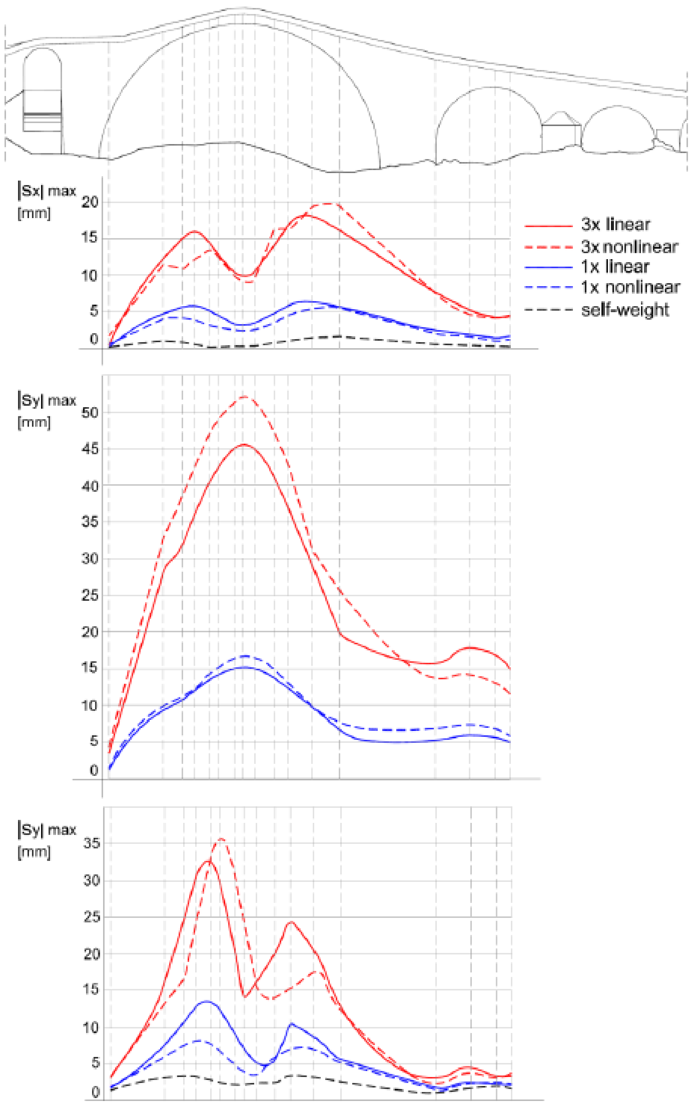
Dynamic Analyses : Transient analysis



Time-histories of ground acceleration components recorded in Fivizzano by FIVI station on June 21, 2013

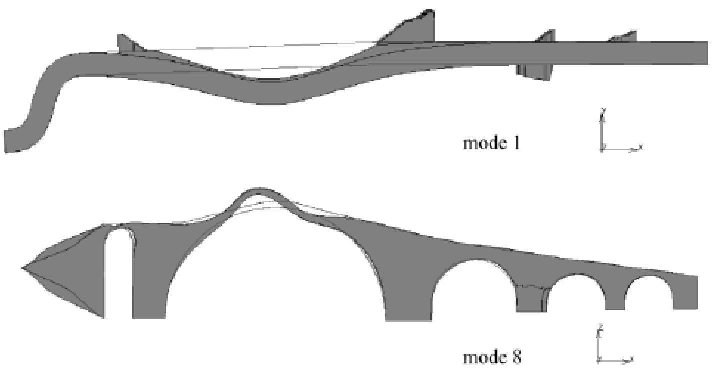
- The dynamic analyses were conducted on Model 2 by applying three history accelerations to the base nodes. The accelerogram employed in the analyses was recorded in Fivizzano station, not too far from the bridge.
- A seismic shaking duration of 12 seconds has been used in all calculations.
- The analysis was conducted using both the accelerogram as originally recorded and then amplified by a factor of 3 to emphasize the results and evaluate the bridge's seismic behavior under extreme condition.
- All analyses were first performed assuming masonry-like behavior with zero tensile strength and limited compressive resistance (3MPa) and then repeated for the elastic case.
- Viscous damping was moreover assumed, with a damping matrix calculated in accordance with the Rayleigh assumption.

Dynamic Analyses : Results

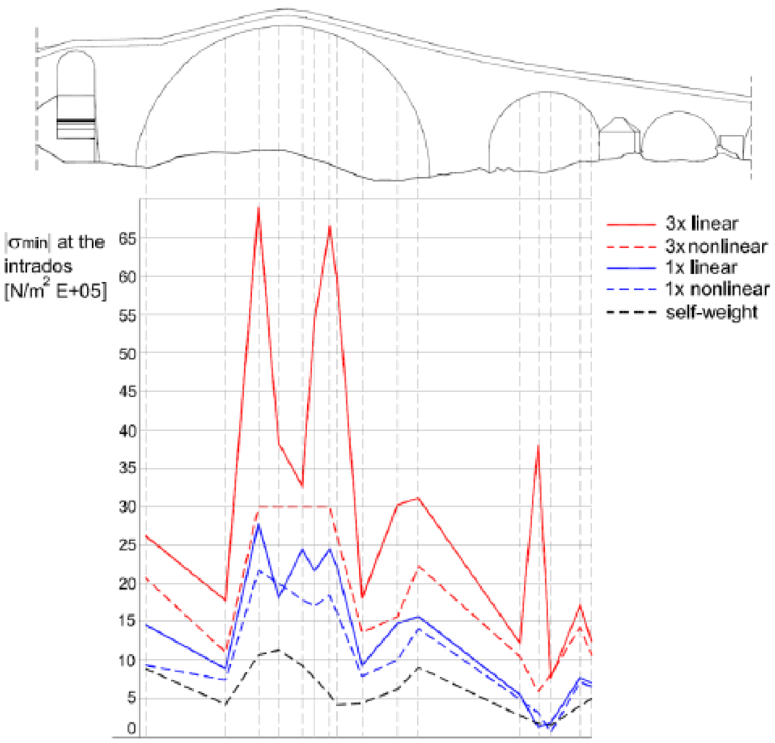


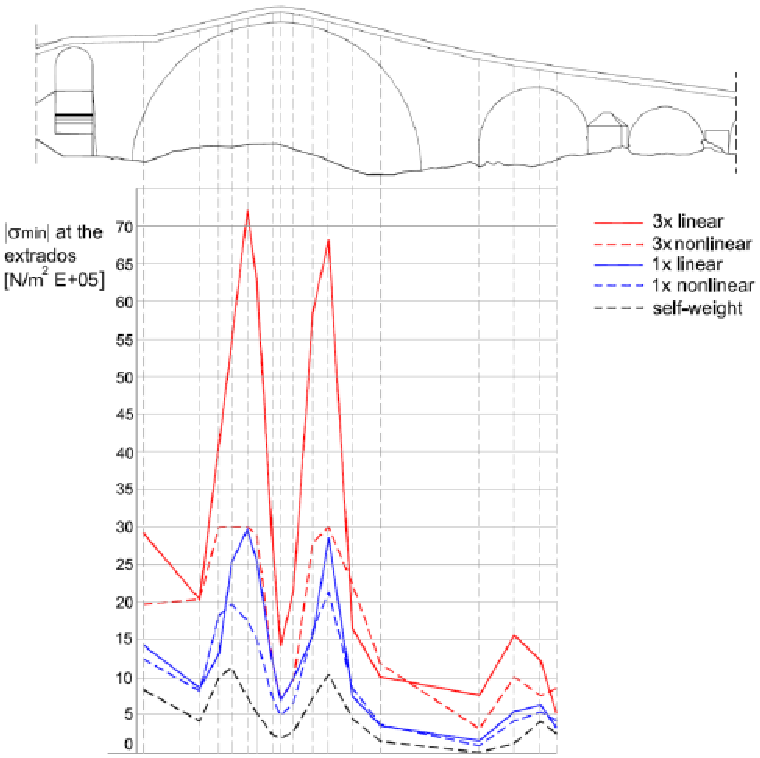
<u>Maximum absolute values of the displacements of the bridge extrados with respect to the base.</u>

The dashed lines represent the nonlinear cases and the continuous lines the linear analysis results. The blue lines refer to the unamplified case, while the amplified case is indicated in red.



Dynamic Analyses : Results

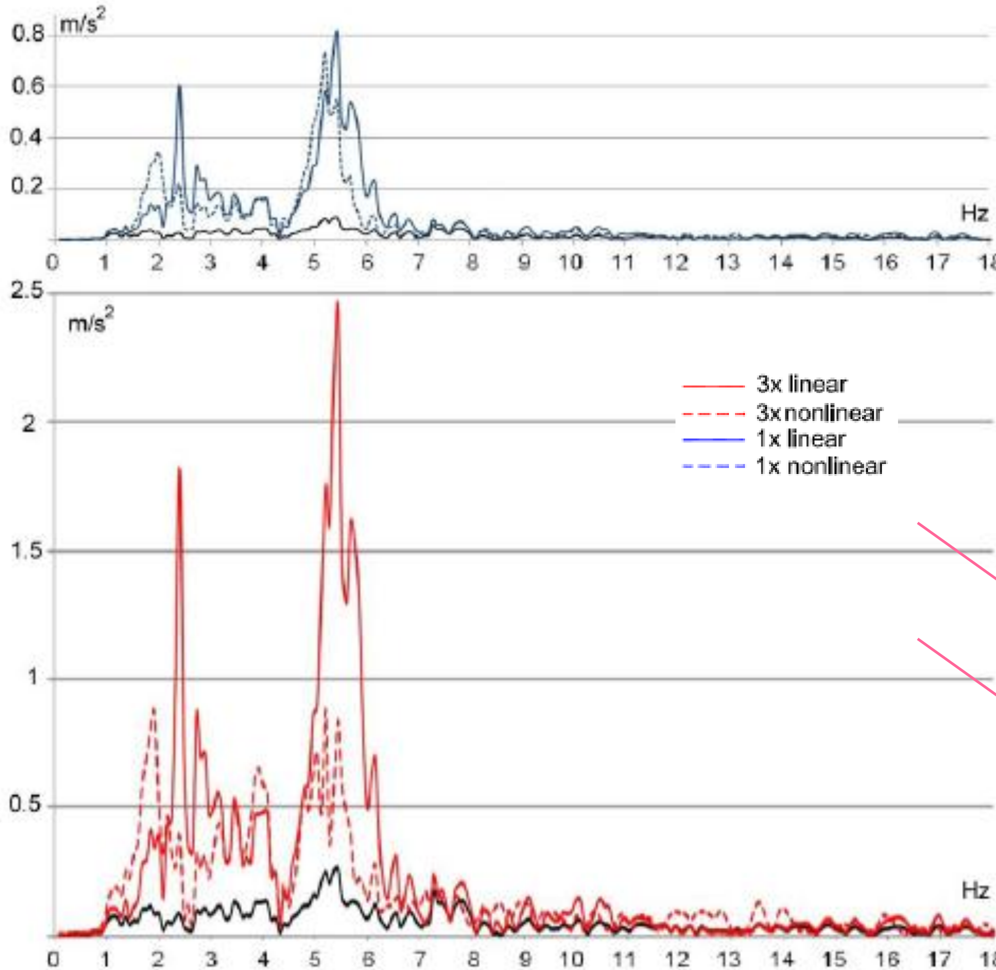




<u>Absolute values of the minimum principal stresses attained at the intrados (left) and extrados (right).</u>

The dashed lines represent the nonlinear cases and the continuous lines represent the linear analysis results. The red line indicates the results for the case with input amplification; the blue lines refer to the unamplified case.

Dynamic Analyses : Results



Discrete Fourier Transform (DFT) of the y-acceleration at the base (black) and the y-acceleration responses for both the linear (continuous) and nonlinear (dashed) cases for the unamplified (blue) and amplified (red) input.

	Brick model (Model 1)				Shell model (Model 2)			
n°	ν [Hz]	mass x	mass y	mass z	ν [Hz]	mass x	mass y	mass z
1	2.18	0.002	**0.19**	5.9E-05	2.19	1.7E-04	**0.22**	2.6E-07
2	3.86	4.9E-04	2.5E-02	3.3E-04	4.09	4.2E-04	0.031	1.7E-06
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8	9.03	**0.22**	8.9E-03	1.4E-04	9.11	**0.23**	0.005	2E-04
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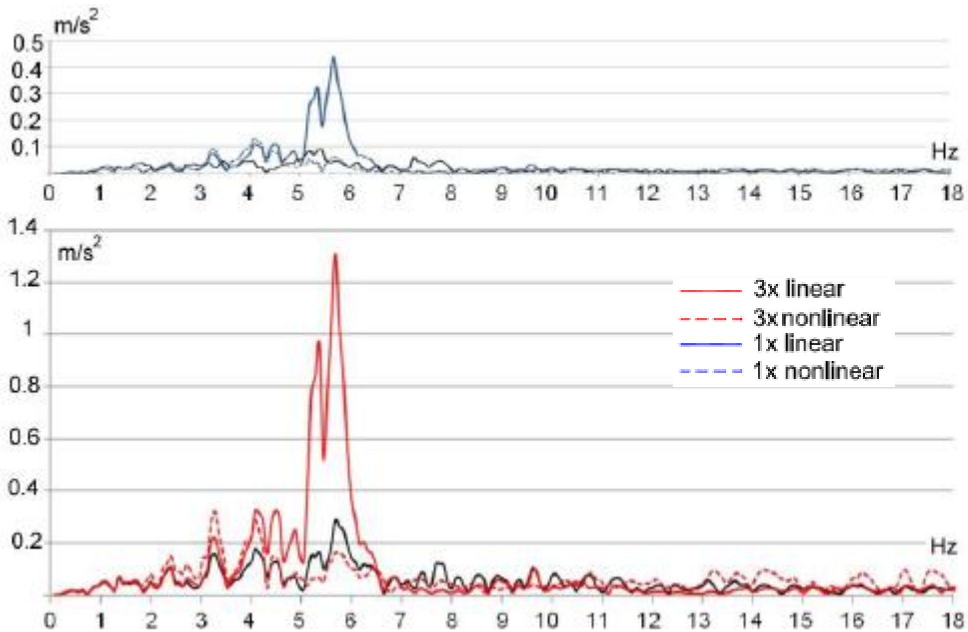
10	9.75	1.6E-05	2.0E-04	**0.12**	10.1	4E-05	0.002	7E-05

Dynamic Analyses : Results



Discrete Fourier Transform (DFT) of the x-acceleration at the base (black) and the x-acceleration responses for both the linear (continuous) and nonlinear (dashed) cases for the unamplified (blue) and amplified (red) input.

	Brick model (Model 1)				Shell model (Model 2)			
n°	ν [Hz]	mass x	mass y	mass z	ν [Hz]	mass x	mass y	mass z
1	2.18	0.002	**0.19**	5.9E-05	2.19	1.7E-04	**0.22**	2.6E-07
2	3.86	4.9E-04	2.5E-02	3.3E-04	4.09	4.2E-04	0.031	1.7E-06
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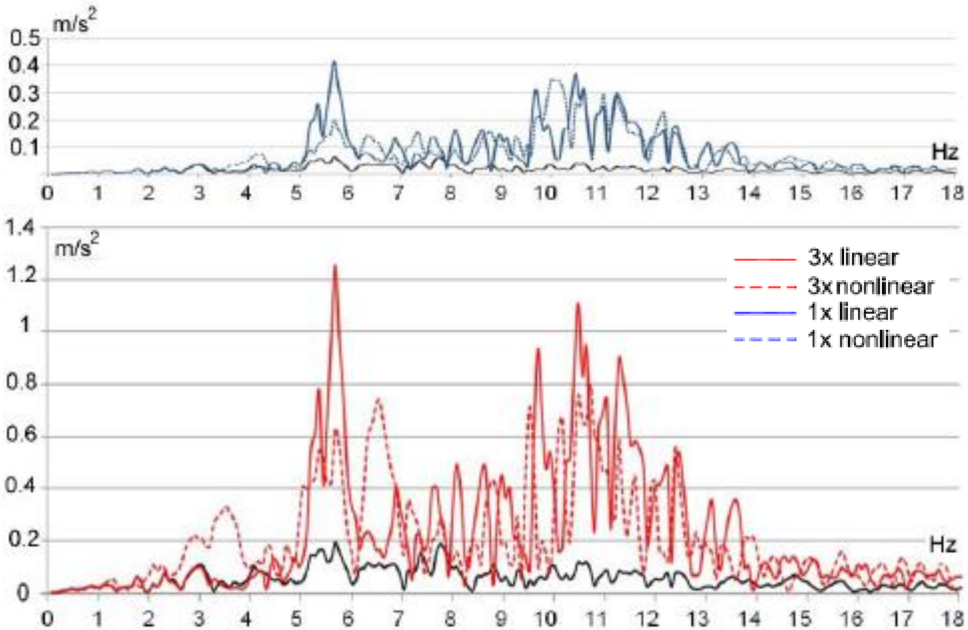
Dynamic Analyses : Results



Discrete Fourier Transform (DFT) of the z-acceleration at the base (black) and the z-acceleration responses for both the linear (continuous) and nonlinear (dashed) cases for the unamplified (blue) and amplified (red) input.

	Brick model (Model 1)				**Shell model (Model 2)**			
n°	ν [Hz]	mass x	mass y	mass z	ν [Hz]	mass x	mass y	mass z
1	2.18	0.002	**0.19**	5.9E-05	2.19	1.7E-04	**0.22**	2.6E-07
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10	9.75	1.6E-05	2.0E-04	**0.12**	10.1	4E-05	0.002	7E-05

Conclusions

- The static and seismic behavior of the Maddalena bridge in Borgo a Mozzano has been studied using **NOSA-ITACA,** a finite element code that models masonry as a nonlinear elastic material with no tensile strength and limited compressive strength.

- The bridge is studied under both its self-weight alone and by applying a base acceleration time history taken from records of an actual earthquake that occurred in the region on June 21st, 2013. Further analyses were also conducted using the original accelerogram amplified by a factor of three.

- The static analysis highlights the good response of the bridge under its self-weight alone.

- The dynamic analyses reveal the differences between the linear and nonlinear responses in term of displacement, stress level and frequency content. The material's nonlinear behavior alters the frequency content of the structural response and the stress state with respect to the linear case.

- **Further** work on the code is necessary to guide interpretation of the results and improve detection of masonry damage. Nonetheless, by neglecting all forms of dissipation apart from viscous damping, the constitutive equation of masonry–like material enables arriving at a generally conservative estimate of the structure's dynamic response.


Thank you for your kind attention
Daniele Pellegrini